Collective Savings Pension Policy in an Economy with Heterogeneity and Informality*

Sistema de pensiones de ahorro colectivo en una economía con heterogeneidad e informalidad

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Abstract

We compare the macroeconomic effects of a fully funded individual defined contribution (IDC) pension scheme, an unfunded pay-as-you-go (PAYG) system, and a collective defined contribution (CDC) regime. Under the latter, contributions of workers from a given cohort are invested in capital markets and repaid to that cohort upon retirement; its collective nature arises from an intragenerational progressive redistributive rule. Our results from an overlapping generations model calibrated for Chile show that the CDC scheme has similar macroeconomic effects as an IDC plan, including a moderate positive effect on the formal labor market, aggregate savings, and output. The PAYG system has negative effects on all these dimensions. Critical for the success of the CDC scheme is conditioning benefits on contributions, to incentivize formal labor status. We conclude that a CDC policy stands as a sustainable alternative for countries with significant labor informality and income inequality.

Key words: *Overlapping generations models; Pension system; Informal labor market; Heterogeneous agents.*

JEL Classification: *E26, E27, H55, J46.*

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Resumen

Se comparan los efectos macroeconómicos de sistemas de pensiones de capitalización individual, reparto y de capitalización colectiva. Bajo este último, las contribuciones de los trabajadores de una cohorte determinada son invertidas en los mercados de capitales y utilizados para pagar las pensiones de esa misma cohorte una vez que esta se retira. Su naturaleza colectiva surge de una regla de redistribución intrageneracional progresiva. Nuestros resultados basados en un modelo de generaciones solapadas calibrado para Chile muestran que este sistema tiene efectos macroeconómicos similares al régimen de capitalización individual, incluyendo un efecto positivo moderado sobre el mercado laboral formal, el ahorro agregado y la actividad. El sistema de reparto tiene efectos negativos en todas estas dimensiones. Central para el desempeño exitoso del sistema de capitalización colectiva es el condicionar pensiones en las contribuciones de los trabajadores, de manera de incentivar el trabajo formal. Concluimos que la implementación de un sistema de capitalización colectiva es una política sostenible para países con significativa informalidad laboral y desigualdad de ingreso.

Palabras clave: *Modelos de generaciones solapadas; Sistema de pensiones; Mercado laboral informal; Agentes heterogéneos.*

Clasificación JEL: *E26, E27, H55, J46.*

1. Introduction

In many countries, demographic changes, economic transformations and social demands for better living conditions put increasing pressure on existing pension regimes. Although these challenges are faced by both developed and developing countries, the latter often need to cope with them in an environment of high-income inequality, significant degrees of labor informality and fiscal sustainability concerns in the midst of ageing populations and changing international interest rates. Given this context, this paper examines the effects of alternative pension system reforms in an economy with large labor heterogeneity in terms of income and employment status. We set up a general equilibrium overlapping generations model (OLG) featuring heterogeneous agents and a dual labor market calibrated to the Chilean economy, which serves as our case study, and we use this model to quantitatively evaluate the long-term macroeconomic effects of alternative pension reforms, comparing individual capitalization with collective pension schemes with a redistributive component.

Collective defined contribution (CDC) schemes are one sort of collective pension policy. Under such a scheme, contributions of workers from a given cohort are pooled into one common fund and invested in capital markets, and the proceeds are repaid to the same cohort upon retirement. This type of scheme has been proposed as an alternative to individual defined contribution (IDC) programs, where contributions are deposited into individual accounts, in order to improve risk sharing. CDC schemes are often considered a "third way" between IDC schemes, where all risks are taken on individually, and defined benefit (DB) schemes such as many pay-as-you-go (PAYG) systems that propose a secure retirement income but are more difficult to maintain, especially in the face of slowing population growth. A similarity of CDC schemes with IDC schemes is that they are fully funded, contrary to unfunded PAYG systems. However, unlike IDC schemes, CDC schemes imply some intragenerational redistribution. The properties of collective pension schemes have been analyzed from different perspectives in the literature, including risk-sharing or stability of participation if participation is voluntary (see Gordon and Varian, 1988; Shiller, 1999; Ball and Mankiw, 2007; Gollier, 2008; Cui et al., 2011; Chen et al., 2016, 2017; Kurtbegu, 2018). However, to our knowledge no existing study has attempted to quantify the macroeconomic effects of such schemes, which are critical to evaluate their overall costs and benefits in comparison to alternative pension policies. In addition, no study has analyzed these issues, nor the design of intragenerational redistributive rules for pension benefits, for economies with large informal labor markets and income heterogeneity, such as in many developing countries.

Hence, in this paper we study the long-term macroeconomic effects of implementing alternative pension system reforms in an economy with a significant informal sector and income heterogeneity. To understand these effects, and to quantitatively evaluate them, we construct an OLG model with specific features relevant for developing and emerging economies. We calibrate the model for Chile, a country whose individual retirement pension accounts system has been a model for many countries.[1] We consider four different pension schemes, all financed through an identical increase of pension contributions taking the form of a payroll tax, that solely differ from each other in the way they treat the additional funds and allocate them among retirees. In particular, we consider two versions of a CDC scheme that differ in whether they make pension benefits depend upon the degree of labor effort during working life,

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[1] Chile has switched to a private retirement accounts system in the early 1980s. Many other Latin American countries have followed the Chilean model. These include (with years of adoption in parentheses): Peru (1993), Colombia (1994), Argentina (1994), Uruguay (1996), Bolivia (1997), Mexico (1997), El Salvador (1998), Costa Rica (2001), the Dominican Republic (2003), Nicaragua (2004), and Ecuador (2004) (see Krasnokutskaya et al., 2018).

i.e., a conditional (C-CDC) scheme and an unconditional (U-CDC) scheme, and compare their macroeconomic performance to an IDC scheme and a PAYG alternative.[2] The collective nature of the CDC schemes arises from a progressive redistributive rule that allocates proportionally more benefits to lower-income workers. These are also the workers with more participation in informality. Conditioning the receipt of benefits on employment status within a redistributive design is the key aspect that incentivizes a strong formalization of labor supply at low-income levels, which are those who make for the largest share of informal work, surpassing the effects found under the IDC plan.

Our results show that the C-CDC scheme has similar macroeconomic effects as an IDC plan, including a moderate positive effect on the formal labor market, which together with the rise of compulsory savings and the capital stock, generates and expansion of output. Moreover, the C-CDC scheme produces a stronger reduction of informality than the IDC plan. While the C-CDC scheme also induces opposite incentives at higher wages, the lower formal sector participation at the low end of the skill distribution delivers an overall increase of labor formality in the economy. The macroeconomic performance of the C- CDC policy is significantly better than that of the PAYG system, which has a strong negative effect on all dimensions. The conditionality of pension benefits upon labor effort under the C-CDC plan is critical for these positive results, as a comparison with the U- CDC scheme shows. Furthermore, the C-CDC plan shares with the IDC alternative the ability to cope with deterioration of the old-age dependency ratio, contrary to the PAYG system.

Structural models, as the one we develop here, are well suited for the analysis of pension policies, as well as for other public policies, for several reasons. First, they allow for consistent exercises, in the sense that if a given policy has, for instance, an impact on the labor supply of households but this effect depends, in turn, on how the policy affects their saving decision, a structural model will be able to capture the interaction between the decisions on both margins, something that cannot be addressed in models where these decisions are taken separately. Among other benefits, this consistency allows understanding the general equilibrium effects of a given policy. For example, changes in agents' propensities to save implied by alternative pension reforms have different effects on capital accumulation in equilibrium, and through this channel, on wages and labor market outcomes. Without a structural model, it would be impossible to determine how these second-round effects operate, or whether they are relevant. In addition, a structural model permits a quantitative evaluation of the effects of alternative policies, as long as its parameters are appropriately calibrated (estimated) to capture the main characteristics of agents and of the relevant markets at play.

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2   All of these options — IDC, CDC and PAYG schemes — have been under discussion
      as part of the proposals to reform the Chilean pension system over the last years.

Since a finite life-time horizon for households is critical to capture the labor, saving and consumption decisions of agents over both a working and a retirement period, we depart from a standard OLG model with three generations. Into that framework we incorporate an informal sector, which can be thought of as home production, and where workers are exempt of paying pension contributions and taxes. The introduction of this dimension responds to the significance that informality has in many developing and emerging economies; in Chile, for example, the informal economy accounts for about a third of total employment. Faced with higher pension contributions, agents will have incentives to switch to informality to avoid paying a perceived tax — an incentive particularly relevant for low-productivity households.[3] We also allow for different skill groups and discount factors, as well as different productivities across sectors, permitting us to better match the income distribution, the observed saving rates, and labor market participation in the economy. These sources of heterogeneity in the model are crucial for understanding the distributional implications of the different pension reforms that we consider, as well as for capturing the differential effects that a given policy can have on distinct agents. Finally, the model features certain small open economy characteristics that allow us, for example, to capture the weighted dependency of pension founds' returns on both the domestic and the international interest rate; following the observation that about 40% of the total founds' resources are invested abroad.[4]

Pension policies work mainly through mandatory contributions and, therefore, their effects depend on the ability of agents to revert forced savings

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[3]   Attanasio et al. (2011), when analyzing the 2008 pension reform in Chile, found that even though pensions increased, the probability of paying into the pension system of workers older than 40 years decreased by 4.1%. Camacho et al. (2014) found that the component corresponding to a 10% tax increase in Colombia in 1993 to finance a health plan led to an increase of 4% in informality.

[4]   Our model features a single investment instrument, precluding agents from choosing among different investment portfolios for their pension funds. In Chile, under the current IDC pension system, agents can opt between 5 alternatives, which differ from each other in their domestic/foreign, fixed/variable-income composition. The implementation of an additional contribution may affect this composition and, thus, the funds' returns. For simplicity and tractability, we abstract from this dimension. However, it is worth noticing that a CDC scheme is not suited for an individual choosing the portfolio composition. It could be assumed that the investment strategy that such a scheme would follow would be similar to the default strategy of the current system, i.e., when agents do not actively choose a portfolio, their funds are invested following an age-dependent default strategy. Evidence suggests that allowing agents to move their funds between portfolio types — under an IDC scheme — is detrimental for the funds' performance; see Contreras et al. (2022). As such, our results may be biased in favor of the IDC scheme. Our main goal is to study the structural differences between the different pension schemes considered. The investment strategies they would follow lie beyond the scope of this paper.

by reducing voluntary ones. In this regard, empirical evidence suggests that agents do face both financial and informational frictions that limit their access to credit or the degree in which they associate current contributions with future pension payments and, consequently, their ability or willingness to reduce private savings. To this end, we include a simple mechanism that proxies for informational frictions that lead to an incomplete internalization of future pension benefits. Akin to the role of borrowing constraints, this mechanism prevents mandatory savings from being completely offset by higher indebtedness throughout working life. Indeed, the literature suggests that, though significant, substitution of mandatory savings is not perfect. This mechanism is crucial for our results, as it makes agents react to pension contributions as if they were, partially, taxes. Attanasio and Rohwedder (2003), exploiting the temporal and cross-sectional variation of three pension reforms, find that this substitution is somewhere between 0.65 and 0.75 for men older than 45, weaker for younger men, and non-existent for basic pensions, which suggest liquidity constraints or lack of information. Attanasio and Brugiviani (2003) find similar results for Italy. Botazzi et al. (2006) further find that substitution is higher for more informed individuals. For Chile, Morandé (1998) finds results in the same line, with a degree of substitution of about 0.5.

Regarding the degree to which agents comprehend pension systems and their own need to save, the literature shows a large dispersion among individuals, and that many workers ignore how their pension plans actually work. Lusardi (1999) shows how household savings in the United States are typically insufficient when retirement arrives. Gustman and Steinmeier (2005), also for the U.S., find that, on average, workers' expectations on their future pensions are not strongly aligned with reality. Other empirical evidence shows how individuals use costly credit sources even though they have access to less expensive credits, or how they simultaneously hold liquid assets with low returns and credit card debts. The explanations for this kind of behavior range from lack of financial education to inconsistent intertemporal preferences; see, for example, Laibson (1997), Angeletos et al. (2001) and Campbell et al. (2011). These reasons are consistent, in turn, with the so-called retirement consumption puzzle (or retirement savings puzzle), which describes the significant drop in consumption after retirement (Attanasio and Weber, 2010). Other explanations include a problem to process information, which is not the result of lack of information and cannot be overcome by public education; see New (1999).

A salient point of this paper is the study of the interaction of the different pension schemes with the informal sector. As mentioned above, the introduction of a new pension system creates incentives for agents to move into informality where they can avoid the payment of those contributions. However, we find that under both the IDC and the C- CDC plans informality actually de-

creases, and that this effect is stronger for the second scheme. This differs from the conclusions presented in Joubert (2015), who finds that rising the pension contribution paid by workers from 10 to 15% in Chile would increase informality by about 10%. The reason for the different results is the framework, in particular, the presence of two features. First, in our model agents internalize — though imperfectly — that the newly introduced pension contributions will translate into future pensions. This has a lower effect on the total net return of formal labor than a plain tax that is used by a government to, say, pay expenses, affecting labor supply decisions less. Second, our general equilibrium model — Joubert (2015) uses a partial equilibrium one — allows for second-round effects that operate expanding the economy and creating incentives to move into formality, whenever the introduction of a pension plan increases total savings. These two differences explain why the ultimate result we find is a reduction of informality under the IDC and the C-CDC plans.

The strand of literature that addresses the effects that social security reforms tend to concentrate on particular aspects, such as job mobility, retirement timing, savings, capital market development, and economic growth; see Thomas and Spataro (2016) for a review of econometric works that consider the effects of pension funds, and Kohl and O'Brien (1998) for a survey on the empirical literature on the effects of PAYG systems, mostly on savings. In this group of studies, Holzmann (1997), using a Solow residual specification of TFP, finds that the 1981 pension reform in Chile had a positive effect on economic growth through the improvement of financial markets. In the same line, Schmidt-Hebbel (1998) concludes that the same reform boosted private investment, the average productivity of capital and TFP. Encina (2013), using the pension reform of 2008 in Chile as a treatment and panel data, finds that the improvement of the pension conditions to the poorest individuals in Chile induced a higher withdrawal from the labor market and less periods contributing to the pension system by these individuals. This is to be expected, as the mechanism determining those agents' pensions depends negatively on their pension level (stemming from their actual contributions). We feature such a mechanism in our U-CDC scheme, and a similar force is at play, though other complementary specifications and initial conditions deliver different results in some dimensions. Davis and Hu (2008), in turn, using a panel of 38 OECD and emerging economies find that the pension-assets- to-GDP ratio has a significant positive effect on growth. Our study contributes to this literature through an analysis of the general equilibrium effects of alternative pension schemes in a model with a dual labor market and income heterogeneity.

Parallel and independent work by Frassi et al. (2019) is closely related to our paper. They study the effects on the labor market and on capital accumulation of three different pension plans: a PAYG system, and two fully funded

ones, including an intragenerational redistributive component. However, we differ from their work in several dimensions: first, we set up a richer calibrated model, providing an empirically realistic quantification of the effects of introducing different pension plans. Second, we show the importance that incorporating informational and/or financial frictions has on the conclusions reached; for example, contrary to their findings, an IDC scheme is no longer neutral on capital, consumption, GDP and the labor market when agents do not fully internalize future pensions. Finally, and most importantly, we consider the effects that pension plans have on informality, a margin that supports a C-CDC-type scheme.

Overall, our results are especially relevant for developing and emerging economies similar to Chile, which present significant labor informality and income inequality. These results show that properly designed CDC schemes may be an economically sustainable and politically viable pension policy in such a context.[5]

The rest of the paper is organized as follows. In the next section we present the details of the model, including the main equations that describe the problems faced by the different agents and their optimal decision rules.[6] In section 3, we discuss the calibration of the model. In section 4, we present the results of the quantitative analysis of the different reform scenarios, together with some robustness analysis. In section 5, we present an exercise to analyze how the different schemes perform when the population growth rate decreases. Finally, section 6 concludes.

2. The Model

The model is based on the neoclassical growth models with overlapping generations following the works of Samuelson (1958) and Diamond (1965). To capture aspects relevant to developing or emerging economies and in particular the Chilean case, we add several features to an otherwise standard OLG model with three generations. These extensions include, first, endogenous labor supply with heterogeneous productivities, as in Brunner (1996) and Somma-

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[5]   A limitation of our model is that it largely abstracts from fiscal policy, since the only fiscal instrument is a tax rate on formal labor income to create a wedge between net and gross income. A richer tax structure, including other instruments such as taxes on firms or consumption, its interaction with informality and its role in the financing of pension systems — partially or totally, directly or through debt — could be a relevant extension. While this is an important topic that deserves attention, it lies outside the scope of this paper. Additionally, for simplicity, we abstract in the model from the current solidarity pillar in Chile.

[6]   A detailed description of the model and the steady state computation is provided in the appendix.

cal (2006), among others. This allows for an asymmetric income distribution among the agents. Second, we introduce heterogeneous discount factors which are decreasing in the income level and that allow us to match the saving rates observed in the data. Third, we add an informal sector based on the works of Busato and Chiarini (2004), Busato et al. (2012) and Orsi et al. (2014), with the purpose of capturing variations along the extensive margin of the labor supply between the formal and informal sectors. The informal sector captures possible unemployment situations associated with self-provision of goods as well as employment that does not pay contributions into the social security system. Fourth, we model a social security policy that captures, in a simplified manner, the current pension system in Chile, which is an individual defined contribution scheme. Fifth, we incorporate a reduced-form mechanism that limits the degree in which agents internalize the benefits of pensions, so as to allow for financial and/or informational frictions, that the different income groups may face. Sixth, we consider a financially open economy, where foreigners own a fraction of the domestic capital stock and where national savers invest abroad. Finally, we also add a series of minor extensions throughout the following sections.

Before deriving the model, let us spend a few lines trying to understand the main mechanism operating in this economy. For this, imagine a two-period economy without any frictions, growth and with infinitely patient agents and a zero interest rate, where agents' optimal consumption plan is to consume the same real amount in both periods. In addition, assume agents receive an endowment of 100 consumption units at the beginning of the first period and must decide how much to save for their second life period, in which they can only consume what they have saved. The optimal decision of these agents is simply to consume 50 units in their first period and to save the other 50 to consume in their second one.

Suppose now that the government introduces mandatory savings of 20 consumption units (i.e., a pension). Since agents still would like to consume the same amount in both their life periods and they know they are saving 20 units through mandatory savings, their optimal response will be to consume 50 units the first period and to save the remaining 30 for the second one. Clearly, all that has happened, because of the introduced pension, is a change in the composition of total savings. Consumption has not changed, nor have total savings. Instead of saving 50 units voluntarily, as in the previous case, agents now have reduced their private savings in 20 units to account for the new mandatory savings. Now, what happens if agents further face financial or informational restrictions? Assume agents only internalize 50% of their future pension, meaning that for each unit they contribute, they believe only 0.5 units will be

paid out to them in the second period of their lives.[7] In this scenario, agents then believe that their future pension — paid out in period 2 — will be of 10 units, not of 20. Therefore, since they still want to consume the same amount in both periods of their lives, they will consume 45 units the first period and save 35 for the second one, which added to the 10 units (perceived) pension, will allow them to consume another 45 units the second period. And that is the core mechanism of the model: agents, as a consequence of not being able to fully internalize their future pensions or not being able to fully undo their mandatory savings, will reduce their consumption during their first life period and, by doing so, increase total savings in the economy. In other words, financial or informational frictions, together with mandatory savings, increase total savings, generating an expansionary effect that operates through an increase in capital accumulation. The following table summarizes the cases illustrated above.

TABLE 1

CONSUMPTION AND SAVINGS EXAMPLE

	No mandatory savings	Mandatory savings	
	No frictions	No frictions	Frictions
Endowment period 1	100	100	100
Consumption period 1	50	50	45
Consumption period 2	50	50	55
Voluntary savings	50	30	35
Mandatory savings	0	20	20
Total savings	50	50	55

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7   This could respond to a lack of understanding of how the system works or lack of trust in the survival of the system among other reasons. Alternatively, if agents were forced to save more than what they optimally wanted — say, in a case where agents have hyperbolic preferences and thus a present bias — we could think of agents facing a borrowing constraint which prevents them from undoing part of those forced savings. This is a realistic assumption insofar borrowing against future pensions is generally not possible.

The different pension systems we will later consider will act mainly through this channel and, in a second degree, by altering labor incentives. Informality in this set up, will play an important role as a means of evading pension contributions. The model, introduced next, will allow us to better understand these mechanisms, how they interact with each other, and ultimately, their relative importance.

2.1 General Assumptions

We assume a perfect foresight economy where agents live for three periods. In each period, lasting T years (where T is later set to 20), there are three different generations alive, a young (y), a middle-aged (m), and an old (o) one, such that members of the generation born in period t are young in period t, middle-aged in $t+1$, and old in $t+2$. Agents work when young and middle-aged and, during that time, they decide how much to spend on consumption and one-period savings. These voluntary savings, in turn, are transformed into physical capital that is rented out to perfectly competitive firms in the formal sector. At the beginning of the following period, the remaining capital — net of depreciation — together with the gained rent is returned back to the agents, and so wealth is transferred intertemporally in the model.[8] Agents retire when old, at which point they only consume, financed by both their savings and their pensions. In addition, workers choose how much to work in the formal and in the informal sector, where the latter is modelled as home production without capital. Agents are further divided into n_l different skill groups denoted by the index $i = 1,\ldots,n_l$, ordered from least skilled ($i = 1$) to most skilled ($i = n_l$). The total population of young workers N_t is assumed to grow at a constant rate n, such that $N_t = N_{t-1}(1+n)$. We assume that the relative proportions of the skill groups are constant over time, such that in period t there are $N_{i,t} = \lambda_i N_t$ workers in group i with $\lambda_i \in (0,1]$ and $\sum_{i=1}^{n_l} \lambda_i = 1$. We now describe the model in more detail, starting with the firms in the formal sector. We then proceed to describe the four alternative pension schemes that we consider and the households' problem, before finalizing with the aggregation.

2.2 Formal Sector Firm

We consider a representative firm that demands capital (K_t) and labor (L_t) to produce a homogeneous good, used for consumption and investment, by means of a standard Cobb-Douglas technology,

$$(1) \qquad Y_t = K_t^\alpha \left(A_t L_t \right)^{1-\alpha}$$

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8   Financial openness will alter this setup slightly.

with $\alpha \in (0,1)$, and where A_t denotes the labor-augmenting technology level which is assumed to grow at a constant rate g, such that $A_t = A_{t-1}(1+g).^9$ The labor input is specified as follows:

$$(2) \qquad L_t = \left(\sum_{i=1}^{n_l} a_i L_{i,t}^{\rho} \right)^{1/\rho}$$

where $L_{i,t}$ denotes the demand for workers from skill group i, the parameters $a_{n_l} \geq a_{n_l-1} \geq \ldots \geq a_1 > 0$ measure the productivities of the different worker types, and $\rho < 1$ determines the elasticity of substitution between the different worker types, which is equal to $1/(1-\rho)$. For $\rho = 1$ the workers from the different skill groups are perfect substitutes and as $\rho \to -\infty$ they become perfect complements. Total labor in period t of skill i, is then

$$(3) \qquad L_{i,t} = L_{i,t}^y + L_{i,t}^m$$

The representative firm maximizes its profits subject to the above three equations, treating the rental rate of capital r_t^k and the skill-specific wages, $W_{i,t}^y$ and $W_{i,t}^m$ as given. Substituting out the constraints, the firm's optimization problem can be written as

$$(4) \quad \max_{K_t, L_{i,t}^y, L_{i,t}^m} K_t^{\alpha} A_t^{1-\alpha} \left(\sum_{i=1}^{n_l} a_i \left(L_{i,t}^y + L_{i,t}^m \right)^{\rho} \right)^{\frac{1-\alpha}{\rho}} - r_t^k K_t - \left(1 + \tau^F\right) \sum_{i=1}^{n_l} W_{i,t}^y L_{i,t}^y + W_{i,t}^m L_{i,t}^m$$

Here $\tau^F \in [0,1]$ denotes the new pension contributions that take the form of a payroll tax paid by the firms. These contributions are the ones financing the different pensions schemes whose effects we analyze. The first-order conditions for this problem are,

$$(5) \qquad K_t : r_t^k = \alpha \frac{Y_t}{K_t}$$

$$(6) \qquad L_{i,t}^y : W_{i,t}^y = a_i W_t \left(\frac{L_t}{L_{i,t}} \right)^{1-\rho}$$

$$(7) \qquad L_{i,t}^m : W_{i,t}^m = a_i W_t \left(\frac{L_t}{L_{i,t}} \right)^{1-\rho}$$

where $W_t = \dfrac{1-\alpha}{1+\tau^F} \dfrac{Y_t}{L_t}$. Equation (5) shows that, at the optimum, capital is rented at its marginal return as is standard, while equations (6) and (7) equate

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9   Throughout, upper-case letters denote variables that are non-stationary, due to population growth, productivity growth, or both, while lower-case letters denote stationary variables.

the salaries of each skill group and age with their respective marginal returns. Note that young and middle- aged workers receive the same wage. In particular, it is worth noticing that salaries increase with the specific productivities of each skill group, a_i, and with the degree of complementarity between the different skill groups (determined by ρ); all salaries decrease with the payroll tax τ^F. One can easily check that the first-order conditions guarantee that the zero-profit condition is satisfied in equilibrium.

2.3 Pension Systems

We assume a social security system with two components. The first component, common to all the alternatives we consider, is financed through a labor income tax on formal workers. Each worker pays a constant fraction, $\tau^W \in [0,1]$, of its formal wage to the government both when young and middle-aged. The government invests these proceeds on each workers behalf and, when the workers retire, they receive their original investment plus the respective accumulated interests. This component accounts, in a simplified way, for the existing mandatory pension system in Chile based on individual defined contributions.

In formal terms, let $S_t^{PW,y} = \sum_{i=1}^{n_l} \tau^W W_{i,t}^y L_{i,t}^y$ and $S_t^{PW,m} = \sum_{i=1}^{n_l} \tau^W W_{i,t}^m L_{i,t}^m$ denote the mandatory social security contributions levied, each period, from young and middle-aged formal workers, respectively. Then, the component of the aggregate pension payments financed through employees' contributions, due in period $t+2$, is defined as

$$(8) \qquad P_{t+2}^W = R_{t+2}\left(S_{t+1}^{PW,m} + R_{t+1}S_t^{PW,y}\right)$$

where $R_t = 1 + (1-\gamma)r_t^k - \delta + \gamma r^\star$ denotes the net return on savings in the economy.[10] Here we introduce financial openness in a simplified manner, by assuming that an exogenous fraction $\gamma \in [0,1]$ of savings is invested abroad at a fixed net interest rate $r^\star$, while the remaining savings are invested in domestic capital with net rate of return $r_t^k - \delta$, where $\delta \in [0,1]$ denotes the depreciation rate of capital. The fraction of the individual pension paid to each worker when retired that is financed through workers' contributions is then given by

$$(9) \qquad P_{i,t+2}^W = R_{t+2}\left(S_{i,t+1}^{PW,m} + R_{t+1}S_{i,t}^{PW,y}\right)$$

Note that τ^W becomes inconsequential for consumption, labor supply and total savings as soon as agents understand how this pension component works

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[10]   We are somewhat flexible with some terminology and refer to $s^{i,j}$ as public savings, where $i = PW, PF$ and $j = y, m$.

and as long as they have no financial constraints, since they can simply modify their voluntary savings in order to offset any mandatory pension contribution they are asked to pay. We relax this assumption later.

The second component of the social security system is financed through a constant payroll tax on employers, τ^F, and can be implemented following one of four alternative schemes considered in this paper:[11]

First, $\mu_1 = 1$, called individual defined contribution (IDC), treats the firms' contributions in the same way as the first component of pensions described above treats the workers' contributions. Under this scheme, we assume that all retirees receive the benefit.

Second, $\mu_2 = 1$, called conditional collective defined contribution (C-CDC), treats the aggregate funds collected from the firms' pension contributions on behalf of each generation in the same way as the previous scheme (and in the same way as in the first component). However, it allocates those funds between the members of each generation, when they retire, by paying higher pensions to those retirees who worked more in the formal sector. This scheme imposes an intragenerational redistribution.

Third, $\mu_3 = 1$, called unconditional collective defined contribution (U-CDC), is similar to the second one, except that the allocation rule among retirees does not depend on how much they worked. Instead, under this scheme, only the retirees of the $i < \bar{i}$ skill groups are entitled to this pension (i.e., the least skilled). Additionally, the pension benefits are decreasing in the first component of the pension, $P_{i,t}^W$. Therefore, this scheme captures part of the spirit of the existing solidarity pillar currently in place in Chile.

Fourth, $\mu_4 = 1$, is a pay-as-you-go (PAYG) system. Under this scheme, all funds collected each period from the firms' contributions, that is, those paid out by the firms on behalf of both young and middle-aged formal workers, are distributed among the existing retirees that very same period, in a way that ensures a constant replacement ratio among the beneficiaries of the pension within each generation.

In formal terms, let us first define the fraction of the aggregate pension found financed by the firms and associated to their young and middle-aged workers as $S_t^{PF,y} = \sum_{i=1}^{n_I} S_{i,t}^{PF,y}$ and $S_t^{PF,m} = \sum_{i=1}^{n_I} S_{i,t}^{PF,m}$, respectively. Here, $S_{i,t}^{PF,y} = \tau^F W_{i,t}^y l_{i,t}^y$ and $S_{i,t}^{PF,m} = \tau^F W_{i,t}^m l_{i,t}^m$ denote the fractions corresponding to each skill group. Then, the component of the aggregate pension payments financed through employers' contributions satisfies,

$$(10) \quad P_{t+2}^F = \left(\mu_1 + \mu_2 + \mu_3\right) R_{t+2} \left(S_{t+1}^{PF,m} + R_{t+1} S_t^{PF,y}\right) + \mu_4 \left(S_{t+2}^{PF,m} + S_{t+2}^{PF,y}\right)$$

---

while the individual pension paid out to each individual when retired can be written as

$$P_{i,t+2}^F = \mu_1 R_{t+2} \left(S_{i,t+1}^{PF,m} + R_{t+1} S_{i,t}^{PF,y} \right)$$

(11)

$$+ \left(\mu_2 \sigma_{i,t+2} + \mu_4 \eta_{i,t+2} \right) P_{t+2}^F + \mu_3 \max \left(0, P_{t+2}^{BS} - \frac{1}{3} P_{i,t+2}^W \right) 1_{i<\bar{i}}$$

where $1_{i<\bar{i}}$ is an indicator function taking the value 1 when $i < \bar{i}$ and 0 otherwise, and where P_{t+2}^{BS} denotes a basic solidarity pension — expressed in per capita terms — defined as

(12)
$$P_{t+2}^{BS} = \left(P_{t+2}^F + \frac{1}{3} \sum_{i=1}^{\bar{i}-1} P_{i,t+2}^W N_{i,t} \right) \frac{1}{\sum_{i=1}^{\bar{i}-1} N_{i,t}}$$

The latter is relevant only for the third scheme ($\mu_3 = 1$). Notice P_{t+2}^{BS} is defined so as to exhaust P_{t+2}^F among all retirees in $t+2$.[12] For the second scheme ($\mu_2 = 1$), we define

(13)
$$\sigma_{i,t+2} = \frac{pts_{i,t+2}}{PTS_{t+2}}$$

as the fraction of the funds of the component financed by the firms' contributions, P_{t+2}^F, that each retiree of skill group i receives. Here $PTS_{t+2} = \sum_{i=1}^{n_I} pts_{i,t+2} N_{i,t}$ are the total aggregated points assigned under this scheme and where the individual points, $pts_{i,t+s}$, are assigned according to the following rule (actuarially fair), which agents know and internalize in their decisions:

$$pts_{i,t+s} = R_{t+2} \left(pts_{i,t+1}^m + R_{t+1} pts_{i,t}^y \right)$$

(14)

$$= R_{t+2} \left\{ \alpha_0 + l_{i,t+1}^m \left(\frac{W_{i,t+1}^m}{\overline{W_{t+1}^m}} \right)^{\alpha_1} + R_{t+1} \left[\alpha_0 + l_{i,t}^y \left(\frac{W_{i,t}^y}{\overline{W_t^y}} \right)^{\alpha_1} \right] \right\}$$

The parameters $\alpha_0 \geq 0$ and $\alpha_1 \in R$ control the redistribution intensity of the rule, and $\overline{W_t^y} = (1/N_t) \sum_{i=1}^{n_I} W_{i,t}^y N_{i,t}$ and $\overline{W_{t+1}^m} = (1/N_t) \sum_{i=1}^{n_I} W_{i,t+1}^m N_{i,t}$

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[12]   Following the logic behind the solidarity pillar in place in Chile until recently, under the third scheme ($\mu_3 = 1$), we assume that those agents for whose self-financed pension, $P_{i,t+2}^W$, is lower than a certain upper bound $P_{i,t+2}^{MAS}$ — from Pensión Máxima con Aporte Solidario in Spanish — are entitled to a pension according to the formula $P_{i,t+2}^F = \max \left\{ 0, \left(P_{t+2}^{MAS} - P_{i,t+2}^W \right) P_{t+2}^{BS} / P_{t+2}^{MAS} \right\}$, where P_{t+2}^{BS} is the highest pension that any agent could receive under this pillar, i.e., when $P_{i,t+2}^W = 0$. As a simplifying assumption, we implement this scheme assuming a constant ratio, $P_{t+2}^{BS} / P_{t+2}^{MAS} = 1/3$.

denote the average salaries of both age groups.[13] For the fourth scheme ($\mu_4 = 1$) we define two auxiliary variables, $\eta_{i,t+2}$ and x_{t+2}, where the first variable denotes the fraction of P_{t+2}^F corresponding to each worker and where the second variable denotes a replacement rate. The latter emerges as the solution to the following system of equations:

$$(15) \qquad \frac{\eta_{i,t+2} P_{t+2}^F}{R_{t+2} W_{i,t+1}^m l_{i,t+1}^m} = x_{t+2} \; \forall i$$

$$(16) \qquad \sum_{i=1}^{n_l} \eta_{i,t+2} N_{i,t} = 1$$

Equation (15) ensures that P_{t+2}^F is distributed such that all retirees get the same replacement rate with respect to their formal middle-aged labor income, while the second equation further guarantees that all funds are used. The solution to this system of equations is given by

$$(17) \qquad \eta_{i,t+2} = \frac{R_{t+2} W_{i,t+1}^m l_{i,t+1}^m}{WL_{t+1}^m}$$

$$(18) \qquad x_{t+2} = \frac{P_{t+2}^F}{WL_{t+1}^m}$$

where $WL_{t+1}^m = R_{t+2} \sum_{i=1}^{n_l} W_{i,t+1}^m N_{i,t}$.[14]

Total and individual pensions paid out each period are the sum of both pension components and are, respectively, given by:

$$(19) \qquad P_t = P_t^W + P_t^F$$

$$(20) \qquad P_{i,t} = P_{i,t}^W + P_{i,t}^F$$

Public savings (in the form of the pension found) are defined as

$$(21) S_t^P = S_t^{PW,m} + R_t S_{t-1}^{PW,y} + S_t^{PW,y} + \left(\mu_1 + \mu_2 + \mu_3 \right) \left(S_t^{PF,m} + R_t S_{t-1}^{PF,y} + S_t^{PF,y} \right)$$

---

13   One can show that in the special case where $g = 0$, implying $W_{i,t}^y / A_t = W_{i,t}^m / A_{t-1}$, the second scheme ($\mu_2 = 1$) with parameters $\alpha_0 = 0$ and $\alpha_1 = 1$ is equivalent to the first scheme of individual defined contribution ($\mu_1 = 1$) since, under these special conditions, one has that $\sigma_{i,t+2} P_{t+2}^F = R_{t+2} \left(S_{i,t+1}^{PF,m} + R_{t+1} S_{i,t}^{PF,y} \right)$. Relaxing the conditions on e and g, i.e., when $e < 1$ and/or $g > 0$, both schemes are only approximately equivalent when α_0 and α_1 are set to 0 and 1 respectively.

14   Depending on the definition of the replacement rate, R_{t+2} could be removed from the above equations and from the definition of WL_{t+1}^m. The presence of the term R_{t+2} is to express income in terms of its value in the period in which pensions are paid.

and the budget constraint of the aggregate pension found in period t is given by

$$(22) \qquad P_t + S_t^P = R_t S_{t-1}^P + \underbrace{S_t^{PW,m} + S_t^{PW,y} + S_t^{PF,m} + S_t^{PF,y}}_{\text{levied taxes}}$$

2.4 Workers of Skill Group I

We address now the problems faced by the households in this economy. Each worker's life-time utility is a function of their consumption when young, $C_{i,t}^y$, middle-aged, $C_{i,t+1}^m$, and old, $C_{i,t+2}^o$, of the labor they supply in the formal sector when young and middle-aged, $l_{i,t}^y$ and $l_{i,t+1}^m$, and of the labor they dedicate to home production, also when young and middle-aged, $h_{i,t}^y$ and $h_{i,t+1}^m$. For $i = 1,\ldots,n_l$, it takes the following form:

$$(23) \qquad \begin{aligned} U_{i,t} &= \frac{\left(C_{i,t}^y\right)^{1-\theta}}{1-\theta} + \beta_i \frac{\left(C_{i,t+1}^m\right)^{1-\theta}}{1-\theta} + \beta_i^2 \frac{\left(C_{i,t+2}^o\right)^{1-\theta}}{1-\theta} \\ &\quad -\Theta_{i,t}^y \frac{\left(l_{i,t}^y + h_{i,t}^y\right)^{\phi} + \kappa\left(h_{i,t}^y\right)^{\phi}}{\phi} - \beta_i \Theta_{i,t+1}^m \frac{\left(l_{i,t+1}^m + h_{i,t+1}^m\right)^{\phi} + \kappa\left(h_{i,t+1}^m\right)^{\phi}}{\phi} \end{aligned}$$

where A $\theta > 0$ is the inverse of the elasticity of intertemporal substitution, $\phi \geq 1$ determines the wage elasticity of labor supply, which equals $1/(\phi - 1)$, $\beta_i \in (0,1)$ is the subjective discount factor of skill group i, and $\kappa > 0$ stands as a specific cost for working at home.[15] The variable $\Theta_{i,t}^j$, with $j = y,m$, is an endogenous preference shifter based on Galí et al. (2012) that is taken as given by the workers and satisfies

$$\Theta_{i,t}^y = \chi_i A_t^{1-\theta}\left(\frac{C_{i,t}^y}{A_t}\right)^{-\theta \upsilon}$$

$$\Theta_{i,t+1}^m = \chi_i A_t^{1-\theta}\left(\frac{C_{i,t+1}^m}{A_t}\right)^{-\theta \upsilon}$$

with $\upsilon \in \{0,1\}$ and $\chi_i > 0$. The purpose of this preference shifter is to allow for an incomplete wealth effect on labor supply. When $\upsilon = 0$, we obtain $\Theta_{i,t}^j = \chi_i$, and, thus, the standard constant relative risk aversion (CRRA) utility function implying a non-zero wealth effect; instead, when $\upsilon = 1$, there is no wealth effect.[16]

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[15]   This specific cost can be justified, for instance, by the lack of health insurance, or by job insecurity.

[16]   In any case, the disutility of work is assumed to grow with the factor $A_t^{1-\theta}$ so that the model has a balanced growth path when $\theta \neq 1$.

Home production is implemented through a decreasing returns to scale production function, with labor as the only input; with $A_t b_i \left(\dfrac{h_t^{\xi-1}}{\eta} \right) a_i$ as the return for both the young and middle-aged workers, and with $b_{n_l} \geq b_{n_l-1} \geq \ldots \geq b_1 > 0$. This specification allows for labor productivity differences between the formal sector and the informal sector, while productivity growth is equal across sectors.

Letting $S_{i,t}^y$ and $S_{i,t}^m$ denote voluntary savings when young and middle-aged, respectively, a given generation's budget constraints for periods t, $t+1$, and $t+2$ satisfy:

$$(24) \qquad C_{i,t}^y = \left(1-\tau-\tau^W\right) W_{i,t}^y l_{i,t}^y + A_t b_i \left(\frac{h_t^{\xi-1}}{\eta} \right) a_i h_{i,t}^y - S_{i,t}^y$$

$$(25) \quad C_{i,t+1}^m = \left(1-\tau-\tau^W\right) W_{i,t+1}^m l_{i,t+1}^m + A_{t+1} b_i \left(\frac{h_{t+1}^{\xi-1}}{\eta} \right) a_i h_{i,t+1}^m + R_{t+1} S_{i,t}^y - S_{i,t+1}^m$$

$$(26) \qquad C_{i,t+2}^o = R_{t+2} S_{i,t+1}^m + P_{i,t+2}$$

where τ is an income tax used for the calibration of the informal labor supply.[17] Following Joubert (2015), we will set ξ so that a 5 percentage points income tax increase yields a 10% increase in informal work. The budget constraints can be combined into the perceived intertemporal budget constraint (IBC) that agents use to make their decisions:

$$\underbrace{C_{i,t}^y + \frac{C_{i,t+1}^m}{R_{t+1}} + \frac{C_{i,t+2}^o}{R_{t+1}R_{t+2}}}_{\textit{Present value of consumption}} =$$

$$(27) \quad \underbrace{(1-\tau-\tau^W)\left(W_{i,t}^y l_{i,t}^y + \frac{W_{i,t+1}^m l_{i,t+1}^m}{R_{t+1}} \right) + A_t b_i \left(\frac{h_t^{\xi-1}}{\eta} \right) a_i h_{i,t}^y + \frac{A_{t+1} b_i \left(\frac{h_{t+1}^{\xi-1}}{\eta} \right) a_i h_{i,t+1}^m}{R_{t+1}}}_{\textit{Present value of labor income}}$$

$$+ \quad \underbrace{\varphi_i \frac{P_{i,t+2}}{R_{t+1}R_{t+2}}}_{\textit{Present value of pension}}$$

Here, the parameter $\varphi_i \in [0,1]$ determines the fraction of the present value of the future pension, $\dfrac{P_{i,t+2}}{R_{t+1}R_{t+2}}$, that agents internalize in their consumption,

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17   Note that the third equation incorporates the simplifying assumption that individuals have finite horizons and, therefore, choose to end up with zero assets when they die (i.e., they make no bequests).

saving and labor decisions. This is a simple, reduced-form strategy to capture two different types of frictions, financial and informational. For instance, φ_i can account for borrowing constraints, meaning that only a fraction of future pensions can be used as collateral. Alternatively, φ_i reflects the ignorance of agents about how the pension system works, capturing, for example, the perception that agents may have of pension contributions as taxes, without any beneficial counterpart. It could also capture, for instance, the confidence agents have on the future promised payments to actually materialize, which is not necessarily a negligible factor, at least in emerging economies. The practical purpose of this specification is to prevent a perfect substitution between mandatory savings — in the form of pension contributions — and voluntary savings, which is achieved for those agents with $\varphi_i < 1$.[18]

Workers, then, face the problem of maximizing their life-time utility (23) subject to (27), (20), (9), (11), (13) ,(14), (17) and the definitions for $S_{i,t}^{k,j}$, for $k = PW, PF$ and $j = y, m$, treating $\Theta_{i,t}^{y}$, $\Theta_{i,t+1}^{m}$, $W_{i,t}^{y}$, $W_{i,t+1}^{m}$, P_{t+2}^{BS}, W_{t}^{y}, W_{t+1}^{m}, PTS_{t+2}, WL_{t+1}^{m}, P_{t+2}^{F}, R_{t+1}, and R_{t+2} as given.[19] Next, we present the first-order conditions associated with this problem. First, the equations for labor supply in the formal sector, $l_{i,t}^{y}$ and $l_{i,t}^{m}$ are given by

$$(28) \qquad \Theta_{i,t}^{y} \left(l_{i,t}^{y} + h_{i,t}^{y} \right)^{\phi-1} = \left(C_{i,t}^{y} \right)^{-\theta} \widetilde{W_{l,t}^{y}}$$

$$(29) \qquad \Theta_{i,t+1}^{m} \left(l_{i,t+1}^{m} + h_{i,t+1}^{m} \right)^{\phi-1} = \left(C_{i,t+1}^{m} \right)^{-\theta} \widetilde{W_{l,t+1}^{m}}$$

where net salaries are defined as

$$(30) \quad \widetilde{W_{l,t}^{y}} = W_{i,t}^{y} \left\{ 1 - \tau - (1-\varphi_i)\tau^{W} + \varphi_i \left[\mu_1 \tau^{F} + \mu_2 \frac{\left(W_{i,t}^{y} \right)^{\alpha_1 - 1}}{\left(\overline{W_{l,t}^{y}} \right)^{\alpha_1}} \frac{P_{t+2}^{F}}{PTS_{t+2}} - \mu_3 \frac{1}{3}\tau^{W} \right] \right\}$$

$$\widetilde{W_{l,t+1}^{m}} = W_{i,t+1}^{m} \left\{ 1 - \tau - (1-\varphi_i)\tau^{W} + \right.$$

$$(31) \qquad \left. \varphi_i \left[\mu_1 \tau^{F} + \mu_2 \frac{\left(W_{i,t+1}^{m} \right)^{\alpha_1 - 1}}{\left(\overline{W_{l,t+1}^{m}} \right)^{\alpha_1}} \frac{P_{t+2}^{F}}{PTS_{t+2}} - \mu_3 \frac{1}{3}\tau^{W} + \mu_4 \frac{P_{t+2}^{F}}{WL_{t+1}^{m}} \right] \right.$$

The above equations show how, at the optimum, agents work up to the point where the disutility of working — both in the formal and informal sector — equals the marginal utility of consuming the perceived net returns on that

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[18]   See the introduction for a discussion of related literature.
[19]   We assume here, for simplicity, that agents do not internalize the effects of their labor efforts on P_{t+2}^{F}. A similar assumption is made in Sommacal (2006).

work, which depends on the pension scheme in place. Second, the equations associated with labor supply in the informal sector, $h_{i,t}^y$ and $h_{i,t}^m$, are given by

$$(32) \quad h_{i,t}^y = \begin{cases} \left[\dfrac{\left(C_{i,t}^y\right)^{-\theta}}{\Theta_{i,t}^y} \left(\dfrac{A_t b_i \left(\dfrac{h_t^{\xi-1}}{\eta} \right) a_i - \widetilde{W_{i,t}^y}}{\kappa} \right) \right]^{\frac{1}{\phi-1}} & \text{if } A_t \left(\dfrac{h_t^{\xi-1}}{\eta} \right) b_i a_i > \widetilde{W_{l,t}^y} \\[2em] 0 & \text{otherwise} \end{cases}$$

$$(33) \quad h_{i,t+1}^m = \begin{cases} \left[\dfrac{\left(C_{i,t+1}^m\right)^{-\theta}}{\Theta_{i,t+1}^m} \left(\dfrac{A_{t+1} b_i \left(\dfrac{h_t^{\xi-1}}{\eta} \right) a_i - \widetilde{W_{i,t+1}^m}}{\kappa} \right) \right]^{\frac{1}{\phi-1}} & \text{if } A_{t+1} \left(\dfrac{h_{t+1}^{\xi-1}}{\eta} \right) b_i a_i > \widetilde{W_{l,t+1}^m} \\[2em] 0 & \text{otherwise} \end{cases}$$

Accordingly, at the optimum, agents spend time working in the informal sector only if the income they obtain from working in that sector is larger than what they perceive they could earn if they were to work in the formal sector.

Finally, we can derive the following consumption equations for $C_{i,t}^y$ and $C_{i,t}^m$:

$$(34) \quad C_{i,t}^y = \frac{\left(1-\tau-\tau^W\right)\left(W_{i,t}^y l_{i,t}^y + \dfrac{W_{i,t+1}^m l_{i,t+1}^m}{R_{t+1}} \right) + A_t b_i \left(\dfrac{h_t^{\xi-1}}{\eta} \right) a_i h_{i,t}^y}{1 + \beta_i^{1/\theta} R_{t+1}^{(1-\theta)/\theta} + \beta_i^{2/\theta}\left(R_{t+1}R_{t+2}\right)^{(1-\theta)/\theta}} +$$

$$(35) \quad \frac{\dfrac{A_{t+1} b_i \left(\dfrac{h_t^{\xi-1}}{\eta} \right) a_i h_{i,t+1}^m}{R_{t+1}} + \varphi_i \dfrac{P_{i,t+2}}{R_{t+1}R_{t+2}}}{1 + \beta_i^{1/\theta} R_{t+1}^{(1-\theta)/\theta} + \beta_i^{2/\theta}\left(R_{t+1}R_{t+2}\right)^{(1-\theta)/\theta}}$$

$$C_{i,t+1}^m = \frac{R_{t+1} S_{i,t}^y + \left(1-\tau-\tau^W\right) W_{i,t+1}^m l_{i,t+1}^m + A_{t+1} b_i \left(\dfrac{h_t^{\xi-1}}{\eta} \right) a_i h_{i,t+1}^m + \varphi_i \dfrac{P_{i,t+2}}{R_{t+2}}}{1 + \beta_i^{1/\theta} R_{t+2}^{(1-\theta)/\theta}}$$

The consumption when old, $C_{i,t+2}^o$, is determined by equation (26). There-fore, young and middle-aged workers consume a fraction of the present value of their perceived total life-time income (which decreases as φ_i decreases, implying a larger friction), and save the rest. When old, the workers simply

consume all their savings plus, if applicable, the pension received from the government.

2.5 Aggregation

We now close the model by specifying a number of definitions and aggregate conditions, that are required to hold in equilibrium. Total formal labor supply of each skill type and cohort needs to equal the corresponding labor demand, which requires

$$(36) \qquad l_{i,t}^{y} N_{i,t} = L_{i,t}^{y}$$

$$(37) \qquad l_{i,t}^{m} N_{i,t-1} = L_{i,t}^{m}$$

for $i = 1,\dots,n_l$.[20] Aggregate voluntary private saving is defined as

$$(38) \qquad S_t = S_t^{y} + S_t^{m}$$

where $S_t^{y} = \sum_{i=1}^{n_l} S_{i,t}^{y} N_{i,t}$ and $S_t^{m} = \sum_{i=1}^{n_l} S_{i,t}^{m} N_{i,t-1}$, while mandatory public saving, S_t^{P}, is defined in (21). Given that only a fraction $1-\gamma$ of total savings, $S_t^{T} = S_t + S_t^{P}$, is invested into capital, while the remaining fraction γ is invested abroad, capital each period satisfies

$$(39) \qquad K_{t+1} = (1-\gamma) S_t^{T} + \gamma_t^{*}$$

Notice that we allow, in addition, for foreigners to own some exogenous fraction of the domestic capital stock, $\gamma_t^{*} = \gamma^{*} A_t N_t$, where $\gamma^{*} \geq 0$.[21] This last assumption, together with the assumption stating that part of domestic savings are invested abroad, amount — as already mentioned — to a simple and stylized strategy to model financial openness in a small open economy as the Chilean one.[22]

Finally, aggregate consumption in period t is defined as

$$(40) \qquad C_t = C_t^{y} + C_t^{m} + C_t^{o}$$

where $C_t^{y} = \sum_{i=1}^{n_l} C_{i,t}^{y} N_{i,t}$, $C_t^{m} = \sum_{i=1}^{n_l} C_{i,t}^{m} N_{i,t-1}$ and $C_t^{o} = \sum_{i=1}^{n_l} C_{i,t}^{o} N_{i,t-2}$

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[20]   Note that $N_{i,t}$ young individuals of type i are born each period t, while, also in period t, there are $N_{i,t-1}$ middle-aged individuals, and $N_{i,t-2}$ old individuals, both of type i.

[21]   We assume γ_t^{*} grows with $A_t N_t$, in order to have a balanced growth path.

[22]   It is possible to show that this way of modelling financial openness, implies that the interest rate differential is decreasing in the level of domestic savings.

are the corresponding aggregate consumption levels for each cohort. And, aggregate taxes amount to

$$T_t = \tau \sum_{i=1}^{n_l} W_{i,t}^y L_{i,t}^y + W_{i,t}^m L_{i,t}^m$$

3. Calibration

The model is calibrated to match a series of statistics of the Chilean economy. Table 2 summarizes the base calibration. The duration of a period, given by parameter T, is set to be 20 years. Therefore, the model implies that agents are retired for 20 years following a working life of 40. We assume there are 5 different skill groups in the economy, i.e., $n_l = 5$, that are representative of the income quantiles.[23] The subjective discount factors of the different skill groups, β_i for $i = 1,\ldots,n_l$, in turn, are calibrated to replicate the average saving rates for each income quantile, which are computed from the Family Budget Survey (Encuesta de Presupuestos Familiares — EPF) from 2012, using the methodology proposed by Madeira (2015, 2018).[24] The matched saving rates are 9%, –0.1%, 4.2%, 8.2% and 17% for the first to fifth quantile, respectively, computed as the average savings during active life. These rates imply an aggregated saving rate, weighted by income, of 7.7%.[25] The social security contribution rate paid by the workers, τ^W, is set at 10%, in line with the defined contribution of the current pension system in Chile. The social security contribution rate paid by the firms on behalf of workers, τ^F, is set to 0% in the initial steady state of the model and it is increased to 5% when the alternative pension reforms are implemented in the different exercises. The parameter that determines the disutility of work, χ_i, is calibrated so as to set relative total work — formal and informal — across ability groups to 0.48, 0.63, 0.74, 0.86, and 1, i.e. implied by the 36%, 47%, 56%, 65%, and 75% participation rates of each quantile (the model has no unemployment).[26]

The annual population growth rate, n, is set to 0.5%, and the technological growth rate corresponding to the growth rate of output per capita, wages and other per capita variables in the model, g, is set to 2%, in line with the

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[23]   Quantile number 1, represents the lowest income group, and, therefore, the less skilled group.

[24]   Source: National Statistics Institute (Instituto Nacional de Estadísticas — INE).

[25]   This number is close to the aggregate saving rate for Chile that was estimated to be between 8.3% and 9.8% since 2010 by the OECD (https://data.oecd.org/hha/household-savings.htm).

[26]   Source: the data is provided by the Ministerio de Desarrollo Social y Familia, Subsecretaría de Evaluación Social, based on information from the Encuesta Casen and the Encuesta Casen en Pandemia 2020. In a robustness exercise total labor participation is set equal accross skill groups; results hold.

average growth trend estimated for Chile until 2050 in the base projection scenario of Albagli et al. (2015).[27] Following the same study, the labor share in production, $1-\alpha$, is set to 52%, which corresponds to the 2008-2013 average of the ratio between salaries paid by the corporate sector and the aggregate value of that sector, net of taxes, according to National Accounts data for Chile.[28] The capital depreciation rate, δ, is set at 4% per year.

TABLE 2

BASELINE CALIBRATION, * QUANTILES $\left(Q_1, Q_2, Q_3, Q_4, Q_5\right)$

Parameter	Description	Value/Target	Source
T	Duration of a period	20 years	-
$\beta_i^{-1/T} - 1$	Annual discount rates	Saving rates	Madeira (2018)
τ^W	Employee contribution rate	10%	Existing rate in Chile
τ^F	Employer contribution rate	$0 \to 5\%$	Exercise
τ	Income tax	30%	-
χ_i	Labor disutility	$l_i = \left[0.48, 0.63, 0.74, 0.86, 1\right]$	CASEN and CASEN en Pandemia 2020
n	Annual population growth rate	0.5%	Albagli et al. (2015)
g	Annual technological growth rate	2%	Albagli et al. (2015)
θ	Intertemp. elasticity of subs., inv	1	Literature
ϕ	Labor supply elasticity, inverse	3	Orsi et al (2014)
α	Capital share	0.48%	Albagli et al. (2015)
$1-(1-\delta)^{1/T}$	Annual depreciation	4%	Litearture
κ	Work at home specific cost.	1.3	informal return between gross-net formal return
ξ	Informal sector prod. fct.	0.53	10% informality increase $\tau : 30 \to 35\%$
η	Informal sector prod. fct	$\left(\dfrac{h_t^{\xi-1}}{\eta}\right) = 1\,in\;first\;s.s.$	Normalization

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[27] This last estimation is based on data and estimations from the Central Bank of Chile, National Socioeconomic Characterization Survey (Encuesta de Caracterización Socioeconómica Nacional — CASEN), the INE and the OECD. Its methodology is based on the production function.

[28] Source: Central Bank of Chile.

φ_i	Friction	$[0.5,0.5,0.5,0.5,0.5]^*$	Literature
υ	Preference shifter	1	Greenwood et al. (1998)
ρ	Labor elasticity of subs. b/ skills	0.33	Ciccone and Peri (2005)
a_5	Productivity, F. sector, skill gr. 5	1	Calibration targets
a_4	Productivity, F. sector, skill gr. 4	0.48	Labor income quantiles
a_3	Productivity, F. sector, skill gr. 3	0.21	(CASEN 2015)
a_2	Productivity, F. sector, skill gr. 2	0.19	
a_1	Productivity, F. sector, skill gr. 1	0.05	$[4.2,9.7,14.5,21.1,50.5]^*$
$a_5 b_5$	Product. Inf. sector, skill gr. 5	0.21	Calibration targets aggregate
$a_4 b_4$	Product. Inf. sector, skill gr. 4	0.09	participation in informal sector
$a_3 b_3$	Product. Inf. sector, skill gr. 3	0.06	and quantile distribution
$a_2 b_2$	Product. Inf. sector, skill gr. 2	0.04	(NESI 2012)
$a_1 b_1$	Product. Inf. sector, skill gr. 1	0.02	$[57,18,18,17,14]^*$

The parameter that determines the specific utility cost of working at home, κ, is set to 1.3, so as to make the return in informality lower than the gross wage and larger than the net wage. The parameter ξ, in turn, is set so that a 5 percentage points income tax increase, yields a 10% increase in informal work following Joubert (2015), and η is set so that $\left(\dfrac{h_t^{\xi-1}}{\eta}\right)$ equals 1 in the first steady state. Joubert (2015) finds that informality increases 10% when a 5 percentage points increase in the pension contribution is introduced in a model for Chile, not as a consequence of an increase of the income tax. However, the model of Joubert (2015) is a partial equilibrium one where the pension contribution behaves more closer to an income tax as the one introduced here, than to a pension contribution. The parameter υ, which determines the size of the wealth effect on labor supply is set to 1. This specification favors the

class of preferences that have a zero wealth effect, similarly to the preferences proposed by Greenwood et al. (1988). Consequently, when analyzing changes in labor supply, we emphasize the role played by wage fluctuations, and do not consider the effects that arise from an intertemporal substitution, with a labor supply schedule that has a positive slope with respect to the wage.

The productivity parameters in the formal sector, a_i for $i = 1, \ldots, n_l$, are calibrated to replicate the distribution of labor income (percentage of total income corresponding to each quantile) as reported by the CASEN 2015 survey.[29]

In turn, the parameters that determine how efficient home production is, b_i, are calibrated so as to obtain an aggregate participation in the informal sector of 25%, and participations, in that same sector, of 57%, 18%, 18%, 17% and 14% for quantiles 1 to 5, respectively. These statistics were computed using data from the New Supplementary Income Survey (Nueva Encuesta Suplementaria de Ingresos — NESI) from 2012, counting as informal workers employees without a contract and self-employed individuals that work at home or on the street, and counting workers with a university education as formal workers.[30] The parameter A, which determines the elasticity of substitution between the different skill groups, is set to 0.33, following Sommacal (2006). This value implies an elasticity of substitution between the different skill types of 1.5, in line with the empirical evidence for the U.S. reported by Ciccone and Peri (2005).

The fraction of domestic savings invested abroad, γ, is set to 40%. This number corresponds to the value of all foreign investment by the Pension Fund Administrators (Administradoras de Fondos de Pensiones — AFP), as a fraction of total assets, according to the data from the Chilean Superintendency of Pensions from August 2016. The foreign interest rate, r^*, is set to 3% annually, which corresponds to the global real return rate assumed by the OECD (2015). In turn, the fraction of foreign investment in Chile, γ^*, is set to 145% of GDP at the initial steady state. This number corresponds to the gross investment position of foreign entities in Chilean banks, other financial institutions, non-financial firms, and households, expressed as a ratio of GDP, according to National Accounts.

Regarding the parameters that determine the financial and/or informational frictions, we will use two different calibrations and complement these with two extreme cases (see the results). The baseline calibration assumes a homogeneous friction where $\varphi_i = 0.5$ for all i, which implies an incomplete degree of substitution between mandatory savings, in the form of pension contributions, and voluntary savings of approximately 0.5, in line with the interna-

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[29] According to this survey the fractions of total income corresponding to each quantile, starting with the first, i.e., skill group number 1, are 4.2%, 9.7%, 14.5%, 21.1%, and 50.5%.

[30] Source: INE.

tional and Chilean evidence (e.g., Attanasio and Brugiavini, 2003; Attanasio and Rohwedder, 2003; Botazzi et al., 2006; Morandé, 1998). This imperfect substitution could be explained by both financial (e.g., because of borrowing constraints derived from limits to the usage of future pensions as collateral) or informational (e.g., because of a partial perception of pension contributions as taxes) frictions. The alternative calibration assumes $\varphi_1 = 0.18$, $\varphi_2 = 0.32$, $\varphi_3 = 0.43$, $\varphi_4 = 0.45$, and $\varphi_5 = 0.50$ which corresponds to the simple average of the number of right answers, for the respective income quantile, of a subset of questions of the Social Protection Survey (Encuesta de Protección Social — EPS) of 2009. This setup corresponds to an interpretation of the friction embodied in φ_i as an informational one.[31] This methodology follows Landerretche and Martínez (2011).

4. COMPARATIVE STATICS

In this section we present the results obtained from the different comparative statics exercises that aim at understanding the effects that the different pension reforms would have on the economy, and in particular the effects of the C-CDC pension scheme. These exercises consist of comparing the initial and final steady states, where the former corresponds to the current state of the Chilean economy with its current pension system, and the latter to the one obtained after a given reform has been implemented. The tables presented show the long-run changes of the main variables of interest — expressed in efficiency adjusted per capita terms — following the introduction of an employer contribution of 5% under the different schemes previously described. The results are obtained using numerical methods that solve for the steady state of the non-linear model.[32]

One of the main uncertainty sources of our model is the financial and/or informational frictions; responsible for the main effects in the model. Therefore, for each pension scheme considered we compute the effects for the baseline scenario, with a degree of internalization of future pension benefits of 50%, constant across skill groups, i.e., $\varphi_i = 0.5 \ \forall i$, together with three alternative scenarios, a case in which internalization is complete, i.e., $\varphi_i = 1 \ \forall i$, a case where there is no internalization of future benefits, i.e., $\varphi_i = 0 \ \forall i$, and a case in which the internalization degree is heterogeneous (see the calibration section). For these last three scenarios the model is re-calibrated to match the same

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[31] Source Centro de Encuestas y Estudios Longitudinales of the Pontificia Universidad Católica de Chile and Subsecretaría de Previsión Social of the Chilean Government.

[32] In particular, we solve for the capital-labor ratio under a specific steady state (which involves solving a non-linear equation), and then rely on using homothopies to bring the model to the desired parametrization.

targets as the ones matched under the baseline model. Finally, we present a robustness scenario, where the fraction of domestic savings invested abroad, γ, is set to 0. For this last case, the model is not re-calibrated, and is left as a robustness exercise.

4.1 Individual Defined Contribution

Table 3 shows the effects of implementing the IDC scheme. We first discuss the case in which there are no frictions, i.e., $\varphi_i = 1 \; \forall i$ and no income tax $\tau = 0$ (case not shown). In that particular case the mechanism is rather simple. As τ^F is increased from 0 to 0.05, wages paid by the firms decrease by 4.8% and pensions increase accordingly.[33] The rest of the economy remains unaltered: as firms discount the new contribution from the workers' wages, workers accept this without changing their labor supply. This happens because workers' total net return for working in the formal sector — what they care about, the sum of the wage and the (discounted) future pension — remains unchanged. There is, however, a composition effect in total savings: the imposed new savings through firms' contributions on behalf of workers are exactly offset by a reduction in the amount of voluntary (private) savings.

This simple mechanism is disrupted when an income tax, τ, is allowed for. A positive τ makes the contributions paid by firms a mechanism to reduce the base on which the income tax paid by workers is computed. This happens because agents do not pay taxes on pensions. Therefore, with a positive income tax, the previous result does change. Agents experience an increase in net formal returns when the new firms' contributions are introduced; inducing an incentive to move into formality (see Table 3, column robustness $\varphi = 1$) and further reducing wages in the sector (relative to the previous case). As a counterpart of the increase in formal work, informality decreases by -2.7%. Finally, though voluntary savings do decrease, mandatory savings increase by more, yielding a marginal increase in total savings that produces, in turn, an increase in capital, which, together with the increase of formal labor, yields an increase of GDP and consumption.

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[33] Note that, $\dfrac{W^{final} - W^{initial}}{W^{initial}} = \dfrac{\dfrac{W^{initial}}{1+\tau^F} - W^{initial}}{W^{initial}} = -0.048$, when $\tau^F = 0.05$.

TABLE 3

LONG-TERM MACROECONOMIC EFFECTS, INDIVIDUAL DEFINED CONTRIBUTION SCHEME.

*: EXPRESSED IN PERCENTAGE POINT CHANGES. CONSUMPTION ENCOMPASSES TWO ROWS, THE UPPER ONE SHOWS THE PERCENTAGE CHANGE WHILE ACTIVE, WHEREAS THE LOWER ONE WHEN

Expressed in percentage changes	Baseline calibration							Friction sensibility			Openness robustness
	Aggregate	Skill groups						φ			γ
		5	4	3	2	1		1	0	Heteroge.	0
Wages	-2.5	-2.6	-2.5	-2.5	-2.5	-2.5		-5.2	0.1	-2.3	-1.6
Formal work	1.4	1.4	1.3	1.3	1.3	1.3		1.8	0.1	1.4	2.2
Informal work	-2.0	-4.2	-2.8	-2.7	-2.6	0.1		-2.7	-0.1	-1.7	-4.0
Formal net income	1.2	1.2	1.2	1.2	1.2	1.3		1.6	0.1	1.0	2.2
Voluntary savings/GGP*	-2.5	-1.3	-0.5	-0.3	-0.2	-0.1		-4.2	-0.8	-2.3	-2.7
Public savings/GDP*	4.2							4.4	4.0	4.1	4.2
Total savings/GDP*	1.7							0.2	3.1	1.8	1.5
Annual interest rate*	-0.1							0.0	-0.2	-0.1	-0.2
Capital	6.4							0.9	10.9	6.8	9.4
Consumption	6.1	1.4	1.6	1.6	1.7	1.5		2.6	7.9	6.3	7.1
		11.3	14.2	16.5	19.4	8.7					
Formal GDP	3.7							1.4	5.2	4.0	5.6

We address now the baseline parametrization, i.e. when $\varphi_i = 0.5 \; \forall i$. Under this scenario, agents view the new pension contribution, τ^F, partially as a tax. They fail to internalize that for every unit contributed, their future pension will increase in that unit plus the accumulated returns it produces until retirement. Instead, they behave as if only half of their contributions translate into future pensions; the other half, as already mentioned, is treated as a tax. Therefore, under this context, the introduction of τ^F induces a reduction of voluntary savings much lower than before and an increase in total savings in the economy; the mechanism illustrated at beginning of the model section. This increase is the main reason behind the significant increase of about 6.4% in the capital stock in the long run. The net formal income increases by 1.2%, making agents work more in the formal sector. At the same time the informal sector becomes less attractive, decreasing 2% in aggregate terms. Although the friction introduces a de facto tax on formal work inducing a negative effect on the labor market, the increase in capital (and the mechanism described above that allows for reducing taxes) generates higher salaries than otherwise. If half of the pension contributions were indeed taxes, and if these were, say, thrown into the sea, the impact would be worse than in the previous case. It is the higher total savings and, consequently, higher capital stock which, finally, push GDP upwards; about 3.7% in the long run.

Setting the friction to the largest possible value, i.e., $\varphi_i = 0 \; \forall i$, we obtain an even more expansionary effect on the economy. However, since agents do not realize that their pension contributions are going to be paid back to them as pensions when they retire, they do not change their labor choices as much as in the baseline scenario. Not allowing domestic households to invest abroad — i.e., setting $\gamma = 0$ — leads to a larger effect on capital, consumption, and output, while the labor market experiences, also, larger movements.

Let us also consider the case in which the friction is heterogeneous. In this case, the less skilled groups will internalize even less of the future pension than they did in the baseline calibration, therefore they will reduce their voluntary savings less, increasing total savings even more, around 1.8%. This will, in turn, lead to an even greater increase in activity and consumption.

4.2 Conditional Collective Defined Contribution

The effects of the C-CDC scheme that conditions on formal work are presented in Table 4. Contrary to the previous scheme, this pension reform imposes a redistribution of the additional funds collected from the firms among the different skill groups of the same generation. Their pension, or more precisely the fraction financed through the firms' contributions, is now proportional to their labor effort and are not weighted by their respective salaries (i.e., we set

$\alpha_0 = 0$ and $\alpha_1 = 0$). This means that the lower skilled agents will face an incentive to move into formality to take advantage of the higher than normal associated pensions, while high skilled agents will experience the opposite.

This scheme generates similar aggregate effects on capital, consumption, and output as the pure individual capitalization scheme addressed previously, while shifting the distribution of formal labor supplied by households more in favor of the lower skill groups. In this way, the effect this scheme has on formality and informality is substantially lar- ger, favoring the formalization of the less skilled groups. Given the relative sizes of the affected groups and the different incentives they are confronted with, aggregate informal work decreases 2.7% .

It should be emphasized that these results hinge significantly on the degree of internalization of future pensions that the different skill groups face. In the case where agents consider their entire contributions as taxes, both schemes — IDC and C-CDC; and U- CDC for that matter — deliver the same aggregate results. This should not be a surprise since agents change their labor and saving decisions solely as a reaction to the perceived tax and they do not observe any of the incentives introduced by the pension. Moreover, while the change in GDP between steady states is higher under the IDC scheme than under the C-CDC one for the baseline calibration, under the heterogeneous calibration this change is lower.

Let us try to explain why this happens. As illustrated before, there are two major forces at play in this model when a new pension contribution is introduced; first, as the degree of internalization decreases, agents undo a smaller fraction of the forced savings imposed by the new pension policy;[34] second, as the degree of internalization increases agents observe the incentives created by the new policy better, consequently modifying their labor and consumption decisions. The first mechanism tends to dominate in the pension schemes we consider here.

First, let us see why changes in savings and activity are higher under the heterogeneous calibration than under the baseline one. The former calibration differs from the latter one in that the lower skilled groups are less able to internalize their future pensions, whereas the most skilled group keeps its degree of internalization constant. As a consequence, under the heterogeneous calibration, the least skilled groups will undo the new mandatory savings less than under the baseline one, while the most skilled groups will save about the same. Therefore, under both pension schemes, aggregate savings and capital increase. Simultaneously, formal labor increases for the most skilled groups

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[34]   This could also be easily thought of in terms of financial restrictions; as borrowing constraints strengthen agents find it harder to borrow against their future pensions, undoing to a lesser extent the introduced mandatory savings.

— mainly due to the general equilibrium effect of higher aggregate savings and capital — and strongly decreases for the lower skilled ones, which now perceive a lower net income for working in the formal sector and therefore, decide to relatively intensify their informal labor.

Let us now try to understand why the increase in activity as a consequence of implementing the IDC scheme is larger than when implementing the C-CDC one under the baseline calibration, whereas this reverses under the heterogeneous one. In the C-CDC case, the most skilled agents reduce their voluntary savings less than under the IDC one because their perceived additional pension, given the newly imposed contribution, ends up being lower under this scheme than under the IDC one. In other words, they cannot allow themselves to reduce their private savings as much, since, as a consequence of the redistributive features of the C-CDC system, for each unit contributed they get paid a lower future pension than under the IDC one. The lower skilled groups, on the other hand, receive higher pensions per unit contributed under the C-CDC scheme than under the IDC one, so they tend to undo a higher fraction of the new mandatory savings under the former scheme than under the latter in an attempt to smooth consumption.

TABLE 4

LONG-TERM MACROECONOMIC EFFECTS, CONDITIONAL COLLECTIVE DEFINED CONTRIBUTION SCHEME.

*: EXPRESSED IN PERCENTAGE POINT CHANGES. CONSUMPTION ENCOMPASSES TWO ROWS, THE UPPER ONE SHOWS THE PERCENTAGE CHANGE WHILE ACTIVE, WHEREAS THE LOWER ONE WHEN RETIRED.

Expressed in percentage changes	Baseline calibration	Skill groups					Friction sensibility φ			Openness robustness γ
	Aggregate	5	4	3	2	1	1	0	Heteroge.	0^{++}
Wages	-2.2	-1.1	-2.5	-3.4	-4.3	-6.6	-4.5	0.1	-1.8	-1.1
Formal work	0.8	-0.9	1.3	2.6	4.1	8.0	0.5	0.1	0.9	1.5
Informal work	-2.7	8.3	-1.9	-6.7	-12.3	-2.6	-3.9	-0.1	-2.2	-4.8
Formal net income	2.4	1.3	2.3	2.9	3.6	6.1	3.6	0.1	1.9	3.5
Voluntary savings/GGP*	0.8	0.5	0.7	0.9	1.0	2.0	0.7	0.1	0.6	1.9
Public savings/GDP*	-2.5	-0.8	-0.6	-0.5	-0.5	-0.2	-4.4	-0.8	-2.2	-2.7
Total savings/GDP*	4.3						4.7	3.9	4.3	4.4
Annual interest rate*	1.8						0.3	3.1	2.0	1.7
Capital	-0.1						-0.0	-0.2	-0.1	-0.2
Consumption	6.6	1.2	2.2	2.9	3.6	3.6	1.0	10.9	7.5	9.8
	6.3	4.5	16.7	26.7	41.3	34.3	2.9	7.9	6.9	7.4
Formal GDP	3.5						0.7	5.2	4.1	5.4

Now, when moving from the baseline to the heterogeneous calibration, the lowest skilled agents under the C-CDC scheme respond by increasing their savings relatively more than the same skilled agents under the IDC scheme. The reason is that when the degree of internalization decreases, under the C-CDC scheme, agents also stop internalizing the redistributive benefits of the scheme. Therefore, savings under the heterogeneous calibration increase relatively more for the C-CDC scheme than for the IDC one, pushing the activity change under the C-CDC system above the change under the IDC one. Other general equilibrium effects are also at play, such as the change in public savings which depends on the changes in wages and in formal employment.

4.3 Unconditional Collective Defined Contribution

Table 5 presents the results obtained from implementing the U-CDC reform. Under this scheme the two most skilled groups of agents are not entitled to an additional pension, even though firms do pay a payroll contribution on their behalf.[35] The remaining skill groups, in turn, receive an additional pension, financed through the firms' contributions, that is decreasing in the first component of their pension, i.e., the one financed through the employees' contributions. The model predicts a much lower output growth between the initial and final steady states, of only 1%. This is partially explained by the lower capital stock growth, about 4.6% in the long run, and, mainly, by the significantly larger reduction in formal labor, -2.3%. This lower aggregate performance ultimately increases informality in the economy, showing how a redistributive mechanism in a collective defined contribution scheme can be a poor design. In fact, informality under this scheme increases considerably, 4.1% in aggregate terms. This pension plan does, notwithstanding, improve the consumption of the three less qualified skill groups in the economy. Notice that for those skill groups that benefit from this scheme, 1 to 3, there is an implicit tax because for each additional unit they receive from their self-financed pension, the fraction of their pension that is financed through firms' contributions decreases by 1/3 units (similarly to the solidarity pillar currently in place in Chile). This is also responsible for the dynamics that end in a larger informality and in a reduction of formal work.

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[35]   This assumption follows the redistribution currently used in the pension scheme in place in Chile, where only two thirds of the retirees, the least skilled ones, are entitled to redistributive benefits.

TABLE 5

LONG-TERM MACROECONOMIC EFFECTS, UNCONDITIONAL COLLECTIVE DEFINED CONTRIBUTION SCHEME.

*: EXPRESSED IN PERCENTAGE POINT CHANGES. CONSUMPTION ENCOMPASSES TWO ROWS, THE UPPER ONE SHOWS THE PERCENTAGE CHANGE WHILE ACTIVE, WHEREAS THE LOWER ONE WHEN RETIRED.

Expressed in percentage changes	Baseline calibration							Friction sensibility			Openness robustness
		Skill groups						φ			γ
	Aggregate	5	4	3	2	1		1	0	Heteroge.	0^{++}
Wages	-1.6	-2.0	-2.1	-0.3	-0.3	-0.4		-3.3	0.1	-0.8	-1.1
Formal work	-2.3	-1.6	-1.6	-4.1	-4.1	-4.0		-4.6	0.1	-1.3	-1.5
Informal work	4.1	2.5	1.5	10.5	10.3	0.4		8.3	-0.1	2.3	3.4
Formal net income	-2.4	-2.0	-2.1	-2.9	-2.9	-3.0		-4.7	0.1	-1.4	-1.8
Voluntary savings/GGP*	-2.6	-0.3	-0.1	-0.7	-0.8	-0.8		-4.5	-0.8	-1.8	-2.8
Public savings/GDP*	4.1							4.3	4.0	4.1	4.2
Total savings/GDP*	1.5							-0.2	3.1	2.2	1.4
Annual interest rate*	-0.1							-0.1	-0.2	-0.2	-0.2
Capital	4.6							-1.6	10.9	7.5	6.8
Consumption	3.5	-3.4	-3.3	4.0	8.0	17.9		-1.5	7.9	5.7	4.1
		-6.9	-7.0	34.4	69.7	172.1					
Formal GDP	1.0							-3.2	5.2	2.9	2.5

4.4 Pay-As-You-Go

Finally, Table 6, shows the effects predicted by the model if a PAYG system would be implemented. This scenario is significantly worse than the previous pension reforms in all dimensions considered. In particular, it is the only system that produces a negative impact on capital, consumption, and output; all of them significant. The decrease in savings, caused by the initial transfer to the old generation leads to a contraction of total savings and, consequently, of the capital stock. This lower capital stock level, in turn, reduces the demand for labor of the firms beyond the direct effect on the labor supply implied by the tax on wages. This reduces formal labor around 4.1%, while increasing informality for all skill groups. As a consequence, GDP in the long run decreases about 4.8%. It is worthwhile mentioning that the rule that determines the distribution of funds is proportional to the contributions made by each agent. This reduces the distortions in the labor market, by strongly linking contributions and future benefits. Other alternative pay-as-you-go schemes should have additional negative effects to the ones predicted here.

Clearly, all modelling and calibration exercises are subject to a substantial degree of uncertainty. In particular, it is difficult to calibrate the financial/informational frictions that make agents internalize only partially the future benefits under the different schemes. Notwithstanding, as it can be observed, the ranking of the different alternatives, in terms of their aggregate effects is relatively robust: the individual defined contribution and the conditional collective defined contribution schemes, have the most positive (or least negative) effects in terms of capital, GDP, and labor, while the unconditional collective defined contribution scheme and, in particular, the pay-as-you-go system, have the (less positive) most negative effects on those variables. In addition, only the first two schemes are able to reduce informality.[36]

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[36]   The collective defined contribution scheme behaves exactly the same as the other first two schemes when the informational and/or financial friction is set to 0 across skill groups, as expected.

TABLE 6
LONG-TERM MACROECONOMIC EFFECTS, UNCONDITIONAL COLLECTIVE DEFINED CONTRIBUTION SCHEME.

*: EXPRESSED IN PERCENTAGE POINT CHANGES. CONSUMPTION ENCOMPASSES TWO ROWS, THE UPPER ONE SHOWS THE PERCENTAGE CHANGE WHILE ACTIVE, WHEREAS THE LOWER ONE WHEN RETIRED.

| Expressed in percentage changes	Baseline calibration						Friction sensibility φ			Openness robustness γ
	Aggregate	Skill groups 5	4	3	2	1	1	0	Heteroge.	0[++]
Wages	-5.4	-5.4	-5.5	-5.5	-5.5	-5.5	-6.2	-4.2	-5.4	-6.3
Formal work	-4.1	-4.1	-4.1	-4.1	-4.1	-4.1	3.3	-4.9	-4.1	4.7
Informal work	6.0	11.8	8.3	8.0	7.9	-0.2	4.6	7.1	6.2	8.3
Formal net income	-5.4	-5.4	-5.5	-5.5	-5.5	-5.5	-6.2	-4.2	-5.3	-6.3
	-2.0	-2.0	-2.1	-2.1	-2.1	-2.1	-0.5	-4.2	-2.3	-3.2
Voluntary savings/GGP*	-0.6	-0.3					-0.8	-0.3	-0.6	-0.4
Public savings/GDP*	-0.7						-0.9	-0.5	-0.7	-0.8
Total savings/GDP*	-1.3						-1.7	-0.7	-1.3	-1.2
Annual interest rate*	0.0						0.1	-0.0	0.0	0.1
Capital	-5.5						-6.3	-3.8	-5.4	-7.8
Consumption	-4.0	-5.7	-5.7	-5.7	-5.7	-5.7	-4.7	-2.9	-3.9	5.0
		-1.9	-1.2	-0.6	0.2	-1.7				
Formal GDP	-4.8						-4.8	-4.4	-4.7	-6.2

5. Demographic Transition

One particularly relevant aspect of pension schemes is their ability to handle population dynamics, such as a decrease in the fertility rate or an increase of life expectancy. In this section we use our model to analyze the first of those situations, simulated by a drop in the population growth rate, and how the different pension schemes can accommodate such a change. We consider a simple steady state transition exercise in which the economy departs form the steady state associated to each pension scheme analyzed, and simulate a drop of n from the calibrated annual 0.5% to 0.25%. In other words, we consider what happens under each of the four alternative pension reforms after they have been fully implemented and the population growth rate drops by half.

The upper-left graph shows the basic population dynamics triggered by the drop in the growth rate n. Remember that each period in the model corresponds to 20 years. In particular, besides the propagation of the shock through the active and total populations, one can observe how as the population growth falls, the amount of passive agents in the economy relative to active workers increases. When the economy stabilizes at its new steady state the ratio of passive individuals relative to the active ones is about 3 percent points higher. Moreover, as population growth slows down, capital becomes relatively more abundant in the economy inducing a decrease in the interest rate. By the same mechanism the relatively more scarcity of workers induce an increase in wages. This increase in wages, in turn, generates a movement of workers from the informal to the formal sector (see upper-left graph in Figure 1). The higher wages together with the increase in formal labor make pension contributions increase. Under the case of the first three schemes, IDC, C-CDC and U-CDC, the drop in n ultimately translates into higher pensions. However, under the PAYG pension alternative, average pensions increase less, as now relatively less workers finance the passive individuals in the economy. It is interesting to note that for all four cases, during the transition, pensions unequivocally decrease, though more strongly under the PAYG scheme. The reason is the decrease in the interest rate and the fact that during the first two periods, the funds used to pay out pensions have been totally or partially in period 2 constituted under the original steady state (when salaries and formal work were lower).

The lower-right graph shows the dynamics of the second component of total pensions, for three pension systems, for the conditional collective defined contribution, for the individual defined contribution, and for the PAYG schemes. We can see how in the case of the PAYG alternative the drop of the second component, the one financed by firms' contributions, is the one pushing total pensions downwards under that scheme. The first component — not shown — behaves in the same way under all four schemes, as is to be expect-

ed. Hence a pension system that is fully articulated as a PAYG system would endure more severe problems, in terms of pensions, in the face of a population growth deceleration.

FIGURE 1

DEMOGRAPHIC TRANSITION
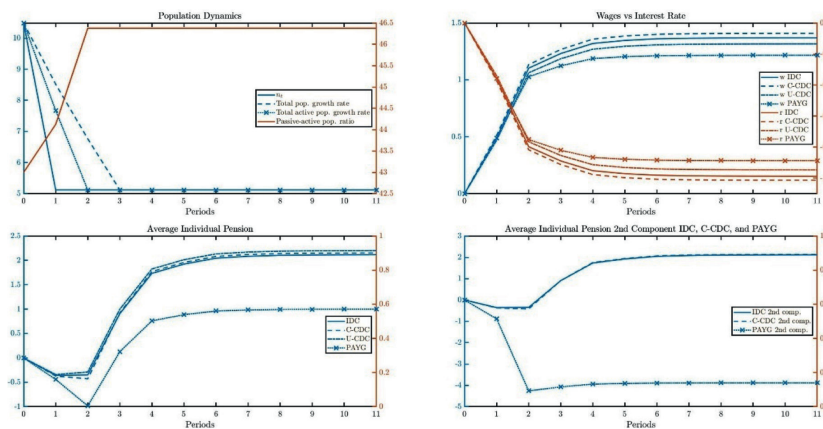
As shown in Figure 2, the slow down in population growth yields larger GDP, consumption and capital stock, for all pension schemes. However, it is worthwhile noticing that the performance of the economy with a PAYG system is clearly dominated by the other three schemes here considered, in particular, by the C-CDC and IDC alternatives.

FIGURE 2

DEMOGRAPHIC TRANSITION
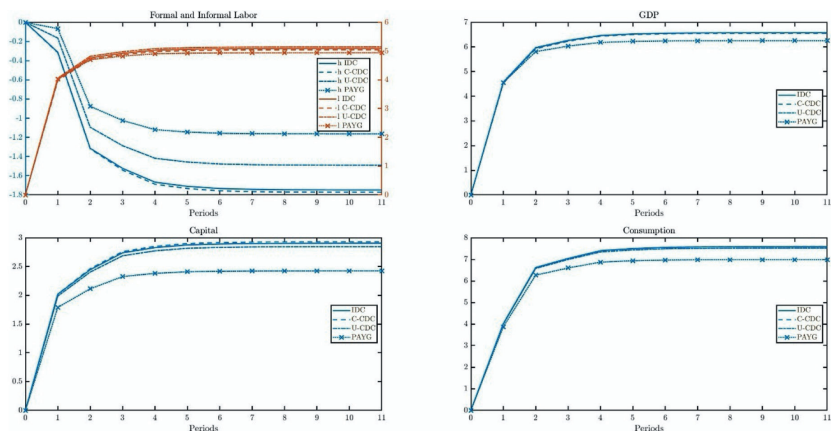
6. Concluding Remarks

In this paper we present a quantitative analysis of the long-term macro-economic effects of implementing alternative pension system reforms in an economy with a significant informal sector. For this, we construct a three-period OLG model with five skill groups, informational and/or financial frictions and labor informality that we calibrate to the Chilean economy. We consider four different pension schemes — all financed through an identical increase of pension contributions taking the form of a payroll tax — that only differ from each other in the way they treat the additional funds and allocate them among retirees. In particular, we consider two versions of a CDC scheme that differ in whether they make pension benefits depend upon the degree of labor effort during working life or not, and compare their macroeconomic performance to an IDC scheme and a PAYG alternative. Conditioning the receipt of benefits on employment status within a redistributive design is the key aspect that incentivizes a strong formalization of labor supply at low income levels, which are those who make the largest share of informal work, surpassing the effects found under the IDC plan. In particular, the informational and/or financial frictions play the main role in the dynamics of our model as they restrict the degree in which agents internalize how pension contributions translate into future pension payments.

The quantitative results suggest that the C-CDC scheme has similar macroeconomic impacts as an IDC plan under the baseline calibration, including a moderate positive effect on the formal labor market, which together with an increase in capital due to the rise of compulsory savings, generates an expansion of output and consumption. With respect to the effect these schemes have on the informal sector, the C-CDC creates stronger incentives to move away from informality among lower-income workers, and since these are precisely the ones more present in informality, we observe a stronger reduction of aggregate informality under this pension plan. This, despite the opposite effect produced among higher-income workers. The U-CDC alternative, in turn, has a negative effect on the labor market, since by making future benefits independent of contributions, low-income agents no longer have an incentive to move into formality. The additional capital is, therefore, no longer complemented by more work, and total output only increases marginally. This reduces employment and formality. The PAYG system constitutes the most adverse scheme of all the ones considered. The important reduction of the capital stock, of around 5.5%, reduces the demand for labor beyond the negative effect of the imposed contribution on firms. Formal employment falls around 4.1% and informality increases about 6%, while GDP and consumption fall 4.8 and 4% respectively.

Finally, we find that like the IDC plan, the C-CDC scheme's solvency is robust to population ageing, the main shortcoming of unfunded pay-as-you-go (PAYG) systems. Hence, our results suggest that a C-CDC scheme may be an economically sustainable and politically viable alternative for countries with significant labor informality and income inequality.

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APPENDIX

MODEL

In what follows we write the stationary model, where variables are expressed in technology adjusted per-capita terms: $c_{i,t}^y = C_{i,t}^y / A_t$, $c_{i,t}^m = C_{i,t}^m / A_{t-1}$, $c_{i,t}^o = C_{i,t}^o / A_{t-2}$, $s_{i,t}^y = S_{i,t}^y / A_t$, $s_{i,t}^m = S_{i,t}^m / A_{t-1}$, $s_{i,t}^{PW,y} = S_{i,t}^{PW,y} / A_t$, $s_{i,t}^{PW,m} = S_{i,t}^{PW,m} / A_{t-1}$, $p_{i,t} = P_{i,t} / A_{t-2}$, $p_{i,t}^F = P_{i,t}^F / A_{t-2}$, $p_{i,t}^W = P_{i,t}^W / A_{t-2}$, $w_{i,t}^y = W_{i,t}^y / A_t$, $w_{i,t}^m = W_{i,t}^m / A_{t-1}$, $l_{i,t} = L_{i,t} / N_{i,t}$, $l_{i,t}^y = L_{i,t}^y / N_{i,t}$, $l_{i,t}^m = L_{i,t}^m / N_{i,t-1}$, $y_t = Y_t / (N_t A_t)$, $l_t = L_t / N_t$, $w_t = W_t / A_t$, $c_t^y = C_t^y / (A_t N_t)$, $c_t^m = C_t^m / (A_{t-1} N_{t-1})$, $c_t^o = C_t^o / (A_{t-2} N_{t-2})$, $s_t = S_t / (A_t N_t)$, $s_t^y = S_t^y / (A_t N_t)$, $s_t^m = S_t^m / (A_{t-1} N_{t-1})$, $s_t^P = S_t^P / (A_t N_t)$, $s_t^{PW,y} = S_t^{PW,y} / (A_t N_t)$, $s_t^{PW,m} = S_t^{PW,m} / (A_{t-1} N_{t-1})$, $s_t^{PF,y} = S_t^{PF,y} / (A_t N_t)$, $s_t^{PF,m} = S_t^{PF,m} / (A_{t-1} N_{t-1})$, $p_t = P_t / (A_{t-2} N_{t-2})$, $p_t^W = P_t^W / (A_{t-2} N_{t-2})$, $p_t^F = P_t^F / (A_{t-2} N_{t-2})$, $\underline{\quad p_t^{BS}} = \underline{P_t^{BS}} / \underline{A_{t-2}}$, $k_t = K_t / (A_{t-1} N_{t-1})$, $pts_t = PTS_t / N_{t-2}$, $w_{i,t}^y = W_{i,t}^y / A_t$, $w_{i,t}^m = W_{i,t}^m / A_{t-1}$

and $wl_t^m = WL_t^m / (N_{t-1} A_{t-1})$. The stationary recursive competitive equilibrium of the model is then the set of sequences,

$$\Big\{ c_{1,t}^y, \ldots, c_{n_l,t}^y, c_{1,t}^m, \ldots, c_{n_l,t}^m, c_{1,t}^o, \ldots, c_{n_l,t}^o, s_{1,t}^y, \ldots, s_{n_l,t}^y, s_{1,t}^m, \ldots, s_{n_l,t}^m,$$

$$s_{1,t}^{PW,y}, \ldots, s_{n_l,t}^{PW,y}, s_{1,t}^{PW,m}, \ldots, s_{n_l,t}^{PW,m}, P_{1,t}, \ldots, P_{n_l,t}, p_{1,t}^W, \ldots, p_{n_l,t}^W, p_{1,t}^F, \ldots, p_{n_l,t}^F,$$

$$l_{1,t}^y, \ldots, l_{n_l,t}^y, l_{1,t}^m, \ldots, l_{n_l,t}^m, h_{1,t}^y, \ldots, h_{n_l,t}^y, h_{1,t}^m, \ldots, h_{n_l,t}^m, w_{1,t}^y, \ldots, w_{n_l,t}^y,$$

$$w_{1,t}^m, \ldots, w_{n_l,t}^m, l_{1,t}, \ldots, l_{n_l,t}, y_t, l_t, w_t, r_t^k, r_t, s_t, s_t^y, s_t^m, s_t^P, s_t^{PW,y}, s_t^{PW,m},$$

$$s_t^{PF,y}, s_t^{PF,m}, p_t, p_t^W, p_t^F, (\mu_2 + \mu_3) p_t^{BS}, k_{t+1}, pts_{i,t}, pts_t, \overline{w_{i,t}^y}, \overline{w_{i,t}^m}, wl_t^m \Big\}_{t=0}^{\infty}$$

such that for given initial values, the following conditions are satisfied for every t :[37]

$$(41) \qquad c_{i,t}^y = (1 - \tau_s) w_{i,t}^y l_{i,t}^y + b_i \left(\frac{h_t^{\xi - 1}}{\eta} \right) a_i h_{i,t}^y - s_{i,t}^y - s_{i,t}^{PW,y}$$

$$(42) \qquad c_{i,t}^m = (1 - \tau) w_{i,t}^m l_{i,t}^m + (1 + g) b_i \left(\frac{h_t^{\xi - 1}}{\eta} \right) a_i h_{i,t}^m + r_t s_{i,t-1}^y - s_{i,t}^m - s_{i,t}^{PW,m}$$

$$(43) \qquad c_{i,t}^o = r_t s_{i,t-1}^m + p_{i,t}$$

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[37]   To cast the system into Dynare syntax, predetermined variables, k_{t+1} and k_t, need to be written as k and $k(-1)$, respectively.

$$(44) \quad c_{i,t}^y = \frac{(1 - \tau - \tau^W)\left(w_{i,t}^y l_{i,t}^y + \dfrac{w_{i,t+1}^m l_{i,t+1}^m}{r_{t+1}}\right)}{1 + \beta_i^{1/\theta} r_{t+1}^{(1-\theta)/\theta} + \beta_i^{2/\theta}(r_{t+1} r_{t+2})^{(1-\theta)/\theta}} +$$

$$\frac{b_i\left(\dfrac{h_t^{\xi-1}}{\eta}\right) a_i h_{i,t}^y + \dfrac{(1+g) b_i \left(\dfrac{h_{t+1}^{\xi-1}}{\eta}\right) a_i h_{i,t+1}^m}{r_{t+1}} + \varphi_i \dfrac{p_{i,t+2}}{r_{t+1} r_{t+2}}}{1 + \beta_i^{1/\theta} r_{t+1}^{(1-\theta)/\theta} + \beta_i^{2/\theta}(r_{t+1} r_{t+2})^{(1-\theta)/\theta}}$$

$$(45) \quad c_{i,t}^m = \frac{r_t s_{i,t-1}^y + \left(1 - \tau - \tau^W\right) w_{i,t}^m l_{i,t}^m + (1+g) b_i \left(\dfrac{h_t^{\xi-1}}{\eta}\right) a_i h_{i,t}^m + \varphi_i \dfrac{p_{i,t+1}}{r_{t+1}}}{1 + \beta_i^{1/\theta} r_{t+1}^{(1-\theta)/\theta}}$$

$$(46) \quad c_t^y = \sum_{i=1}^{n_l} c_{i,t}^y \lambda_i$$

$$(47) \quad c_t^m = \sum_{i=1}^{n_l} c_{i,t}^m \lambda_i$$

$$(48) \quad c_t^o = \sum_{i=1}^{n_l} c_{i,t}^o \lambda_i$$

$$(49) \quad s_{i,t}^{PW.y} = \tau^W w_{i,t}^y l_{i,t}^y$$

$$(50) \quad s_{i,t}^{PW.m} = \tau^W w_{i,t}^m l_{i,t}^m$$

$$(51) \quad p_{i,t} = p_{i,t}^W + p_{i,t}^F$$

$$(52) \quad p_{i,t}^W = r_t \left(s_{i,t-1}^{PW.m} + r_{t-1} s_{i,t-2}^{PW.y}\right)$$

$$(53) \quad p_{i,t}^F = \mu_1 r_t \tau^F \left(w_{i,t-1}^m l_{i,t-1}^m + r_{t-1} w_{i,t-2}^y l_{i,t-2}^y\right) + \mu_2 \frac{pts_{i,t}}{pts_t} p_t^F$$

$$+ \mu_3 \left[p_t^{BS} - \frac{1}{3} r_t \tau^W \left(w_{i,t-1}^m l_{i,t-1}^m + r_{t-1} w_{i,t-2}^y l_{i,t-2}^y\right)\right] 1_{i<\bar{i}} + \mu_4 \frac{r_t w_{i,t-1}^m l_{i,t-1}^m}{w l_{t-1}^m} p_t^F 1_{i<\bar{i}}$$

$$(54) \quad p_t^{BS} = \left(r_t \left(s_{t-1}^{PF.m} + r_{t-1} s_{t-2}^{PF.y}\right) + \frac{1}{3} \sum_{i=\bar{i}+1}^{n_l} p_{i,t}^W \lambda_i\right) \frac{1}{\sum_{i=\bar{i}+1}^{n_l} \lambda_i}$$

$$pts_{i,t} = r_t \left(a_0 + l_{i,t-1}^m \left(\frac{w_{i,t-1}^m}{w_{t-1}^m}\right)^{a_1} + r_{t-1} \left[a_0 + l_{i,t-2}^y \left(\frac{w_{i,t-2}^y}{w_{t-2}^y}\right)^{a_1}\right]\right)$$

$$(55) \qquad pts_t = \sum_{i=1}^{n_l} \lambda_i \, pts_{i,t}$$

$$(56) \qquad \begin{aligned} &\chi_i^y \left(l_{i,t}^y + h_{i,t}^y \right)^{\phi-1} = \left(c_{i,t}^y \right)^{-\theta(1-\upsilon)} w_{i,t}^y \\ &\left\{ 1 - \tau - \left(1-\varphi_i\right)\tau^W + \varphi_i \left[\mu_1 \tau^F + \mu_2 \frac{\left(w_{i,t}^y\right)^{a_1-1}}{\left(\overline{w_t^y}\right)^{a_1}} \frac{p_{t+2}^F}{pts_{t+2}} - \mu_3 \frac{1}{3}\tau^W 1_{i<\bar{i}} \right] \right\} \end{aligned}$$

$$(57) \qquad \begin{aligned} &\chi_i^m \left(l_{i,t}^m + h_{i,t}^m \right)^{\phi-1} = \left(c_{i,t}^m \right)^{-\theta(1-\upsilon)} w_{i,t}^m \\ &\left\{ 1 - \tau - \left(1-\varphi_i\right)\tau^W + \varphi_i \left[\mu_1 \tau^F + \mu_2 \frac{\left(w_{i,t}^m\right)^{a_1-1}}{\left(\overline{w_t^m}\right)^{a_1}} \frac{p_{t+1}^F}{pts_{t+1}} - \mu_3 \frac{1}{3}\tau^W 1_{i<\bar{i}} + \mu_4 \frac{p_{t+1}^F}{wl_t^m} 1_{i<\bar{i}} \right] \right\} \end{aligned}$$

$$(58) \qquad h_{i,t}^y = \max \left\{ 0, \left[\frac{\dfrac{\left(c_{i,t}^y\right)^{-\theta(1-\upsilon)}}{\chi_i^y}}{\dfrac{b_i\left(\dfrac{h_t^{\xi-1}}{\eta}\right)a_i - w_{i,t}^y \left\{ 1 - \tau - \left(1-\varphi_i\right)\tau^W + \varphi_i \left[\mu_1 \tau^F + \mu_2 \dfrac{\left(w_{i,t}^y\right)^{a_1-1}}{\left(\overline{w_t^y}\right)^{a_1}} \dfrac{p_{t+2}^F}{pts_{t+2}} -\mu_3 \dfrac{1}{3}\tau^W 1_{i<\bar{i}} \right] \right\}}{\kappa}} \right]^{\frac{1}{\phi-1}} \right\}$$

$$(59) \qquad h_{i,t}^m = \max \left\{ 0, \left[\frac{\dfrac{\left(c_{i,t}^m\right)^{-\theta(1-\upsilon)}}{\chi_i^m}}{\dfrac{(1+g)b_i\left(\dfrac{h_t^{\xi-1}}{\eta}\right)a_i - w_{i,t}^m \left\{ 1 - \tau - \left(1-\varphi_i\right)\tau^W + \varphi_i \left[\mu_1 \tau^F + \mu_2 \dfrac{\left(w_{i,t}^m\right)^{a_1-1}}{\left(\overline{w_t^m}\right)^{a_1}} \dfrac{p_{t+1}^F}{pts_{t+1}} -\mu_3 \dfrac{1}{3}\tau^W 1_{i<\bar{i}} + \mu_4 \dfrac{p_{t+1}^F}{wl_t^m} 1_{i<\bar{i}} \right] \right\}}{\kappa}} \right]^{\frac{1}{\phi-1}} \right\}$$

$$(60) \qquad w_{i,t}^{y} = a_i w_t \left(\frac{l_t}{\lambda_i l_{i,t}} \right)^{1-\rho}$$

$$(61) \qquad w_{i,t}^{m} = \left(1+g\right) a_i w_t \left(\frac{l_t}{\lambda_i l_{i,t}} \right)^{1-\rho}$$

$$(62) \qquad l_{i,t} = l_{i,t}^{y} + \frac{l_{i,t}^{m}}{1+n_t}$$

for $i = 1,...,n\_\{l\}$. And, also,

$$(63) \qquad \overline{w_t^{y}} = \sum_{i=1}^{n_l} w_{i,t}^{y} \lambda_i$$

$$(64) \qquad \overline{w_t^{m}} = \sum_{i=1}^{n_l} w_{i,t}^{m} \lambda_i$$

$$(65) \qquad wl_t^{m} = r_{t+1} \sum_{i=\bar{i}+1}^{n_l} w_{i,t}^{m} l_{i,t}^{m} \lambda_i$$

$$(66) \qquad y_t = \left(\frac{k_t}{\left(1+g\right)\left(1+n_t\right)} \right)^{\alpha} l_t^{1-\alpha}$$

$$(67) \qquad l_t = \left(\sum_{i=1}^{n_l} a_i \left(\lambda_i l_{i,t} \right)^{\rho} \right)^{1/\rho}$$

$$(68) \qquad w_t = \frac{1-\alpha}{1+\tau^{F}} \frac{y_t}{l_t}$$

$$(69) \qquad t_t = \tau \sum_{i=1}^{n_l} w_{i,t}^{y} l_{i,t}^{y} + \frac{w_{i,t}^{m} l_{i,t}^{m}}{\left(1+g\right)\left(1+n_t\right)}$$

$$(70) \qquad r_t^{k} = \alpha \frac{y_t}{k_t} \left(1+g\right)\left(1+n_t\right)$$

$$(71) \qquad r_t = 1 + \left(1-\gamma\right)\left(r_t^{k} - \delta\right) + \gamma r*$$

$$(72) \qquad s_t = s_t^{y} + \frac{s_t^{m}}{\left(1+g\right)\left(1+n_t\right)}$$

$$(73) \qquad s_t^{y} = \sum_{i=1}^{n_l} s_{i,t}^{y} \lambda_i$$

$$(74) \qquad s_t^{m} = \sum_{i=1}^{n_l} s_{i,t}^{m} \lambda_i$$

$$(75) \quad s_t^{P} = \frac{s_t^{PW,m} + r_t s_{t-1}^{PW,y}}{\left(1+g\right)\left(1+n_t\right)} + s_t^{PW,y} + \left(\mu_1 + \mu_2 + \mu_3\right) \left[\frac{s_t^{PF,m} + r_t s_{t-1}^{PF,y}}{\left(1+g\right)\left(1+n_t\right)} + s_t^{PF,y} \right]$$

$$(76) \qquad\qquad p_t = p_t^W + p_t^F$$

$$(77) \qquad p_t^F = \left(\mu_1 + \mu_2 + \mu_3\right) r_t \left(s_{t-1}^{PF,m} + r_{t-1} s_{t-2}^{PF,y}\right) +$$
$$\mu_4 \left(\left(1+g\right)\left(1+n_t\right) s_t^{PF,m} + \left(1+g\right)^2 \left(1+n_t\right)\left(1+n_{t-1}\right) s_t^{PF,y}\right)$$

$$(78) \qquad\qquad p_t^W = r_t \left(s_{t-1}^{PW,m} + r_{t-1} s_{t-2}^{PW,y}\right)$$

$$(79) \qquad\qquad s_t^{PW,y} = \sum_{i=1}^{n_l} \tau^W \lambda_i w_{i,t}^y l_{i,t}^y$$

$$(80) \qquad\qquad s_t^{PW,m} = \sum_{i=1}^{n_l} \tau^W \lambda_i w_{i,t}^m l_{i,t}^m$$

$$(81) \qquad\qquad s_t^{PF,y} = \sum_{i=1}^{n_l} \tau^F \lambda_i w_{i,t}^y l_{i,t}^y$$

$$(82) \qquad\qquad s_t^{PF,m} = \sum_{i=1}^{n_l} \tau^F \lambda_i w_{i,t}^m l_{i,t}^m$$

$$(83) \qquad\qquad k_{t+1} = \left(1-\gamma\right)\left(s_t + s_t^P\right) + \gamma^*$$

STEADY STATE

Let variables without time subscript denote steady state values. We solve for the steady state by means of numerical methods, using as starting values for the numerical solver the analytical steady state solution for the special case where $\tau^W = \tau^F = 0$ (note that imposing $\tau^F = 0$ is equivalent to $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 0$, $\chi_i^y = \chi_i^m$ for all i, $\beta_i = \beta_j$ for all i and j, $\theta = \rho = \upsilon = 1$ and $\gamma = \gamma^* = 0$. From (46) to (48)

$$c_i^o = r s_i^m = r \left(w_i^m l_i^m + \left(1+g\right) b_i \left(\frac{h^{\xi-1}}{\eta}\right) a_i h_i^m + r s_i^y - c_i^m\right) =$$
$$r \left(w_i^m l_i^m + \left(1+g\right) b_i \left(\frac{h^{\xi-1}}{\eta}\right) a_i h_i^m + r \left(w_i^y l_i^y + e b_i \left(\frac{h^{\xi-1}}{\eta}\right) a_i h_i^y - c_i^y\right) - c_i^m\right)$$

and we also have from (49) through (53) and (75) through (82) that

$$(84) \qquad\qquad s_i^{PW,y} = s_i^{PW,m} = p_i^W = p_i^F = p_i = 0$$

for $i = 1,\ldots,n_l$, and that

(85) $\qquad s^P = s^{PW,y} = s^{PW,m} = s^{PF,y} = s^{PF,m} = p = p^W = p^F = p^{BS} = 0$

From (41) through (43) and the previous results,

$$c_i^o \overset{(43)}{\triangleq} rs_i^m \overset{(42)}{\triangleq} r\left(w_i^m l_i^m + (1+g)b_i \left(\frac{h^{\xi-1}}{\eta} \right) a_i h_i^m + rs_i^y - c_i^m \right)$$

$$\overset{(41)}{\triangleq} r\left(w_i^m l_i^m + (1+g)b_i \left(\frac{h^{\xi-1}}{\eta} \right) a_i h_i^m + rw_i^y l_i^y + rb_i \left(\frac{h^{\xi-1}}{\eta} \right) a_i h_i^y - rc_i^y - c_i^m \right)$$

$\Leftrightarrow$

$$c_i^o + rc_i^m + r^2 c_i^y =$$
$$r\left(w_i^m l_i^m + (1+g)b_i \left(\frac{h^{\xi-1}}{\eta} \right) a_i h_i^m + rw_i^y l_i^y + rb_i \left(\frac{h^{\xi-1}}{\eta} \right) a_i h_i^y \right)$$

We also have that,

(86) $\qquad\qquad\qquad\qquad c_i^m = r\beta_i c_i^y$

and

$$c_i^o = r\beta_i c_i^m$$

therefore,

$$c_i^o = \beta_i r c_i^m = \left(\beta_i r \right)^2 c_i^y$$

Combining both results, we obtain that

(87) $\qquad c_i^y = \dfrac{w_i^m l_i^m + (1+g)b_i \left(\frac{h^{\xi-1}}{\eta} \right) a_i h_i^m + rw_i^y l_i^y + rb_i \left(\frac{h^{\xi-1}}{\eta} \right) a_i h_i^y}{\left(\beta_i^2 + \beta_i + 1 \right)r}$

From (60), and (61), and since $\rho = 1$,

$$w_i^y = a_i w$$
$$w_i^m = (1+g)a_i w$$

Thus,

(88) $\qquad\qquad\qquad\qquad w_i^y = \dfrac{w_i^m}{1+g}$

From (58) and (59), and as long as $\chi_i^y = \chi_i^m$, $\theta = 1$,

$$(89) \qquad h_i^y = \max\left\{ \left[a_i \frac{\left(b_i \left(\dfrac{h^{\xi-1}}{\eta} \right) - w \right)}{\chi_i^y \kappa} \right]^{\frac{1}{\phi-1}}, 0 \right\}$$

$$(90) \qquad h_i^m = \max\left\{ \left[(1+g) a_i \frac{\left(b_i \left(\dfrac{h^{\xi-1}}{\eta} \right) - w \right)}{\chi_i^m \kappa} \right]^{\frac{1}{\phi-1}}, 0 \right\}$$

if $b_i < 0$ we have that,

$$h_i^y = 0 = h_i^m$$

else,

$$h_i^y = \left[a_i \frac{\left(b_i \left(\dfrac{h^{\xi-1}}{\eta} \right) - w \right)}{\chi_i^y \kappa} \right]^{\frac{1}{\phi-1}}$$

$$= \left(\frac{1}{1+g} \right)^{\frac{1}{\phi-1}} \left[(1+g) a_i \frac{\left(b_i \left(\dfrac{h^{\xi-1}}{\eta} \right) - w \right)}{\chi_i^m \kappa} \right]^{\frac{1}{\phi-1}}$$

$$= \left(\frac{1}{1+g} \right)^{\frac{1}{\phi-1}} h_i^m$$

which holds also for $b_i \geq 0$. We further choose b_i such that

$$(91) \qquad h_i^y = \frac{l_i^y w}{b_i \left(\dfrac{h^{\xi-1}}{\eta} \right)}$$

and

$$(92) \qquad h_i^m = \frac{l_i^m w}{b_i \left(\dfrac{h^{\xi-1}}{\eta} \right)}$$

In addition, assuming that such a b_i is constant over i, we have that,

$$(93) \qquad c_i^y = \frac{(1+g)l_i^m + rl_i^y}{\left(\beta_i^2 + \beta_i + 1 \right) r} 2a_i w$$

and

$$(94) \qquad c_i^m = \beta_i \frac{(1+g)l_i^m + rl_i^y}{\beta_i^2 + \beta_i + 1} 2a_i w$$

From (56) and (57),

$$\left(l_i^y \right)^{\phi-1} = \frac{a_i w}{\chi_i^y} \frac{1}{1 + \dfrac{w}{b_i \left(\dfrac{h^{\xi-1}}{\eta} \right)}}$$

$$(95) \qquad \chi_i^m \left(l_i^m \right)^{\phi-1} = (1+g) a_i w \frac{1}{1 + \dfrac{w}{b_i \left(\dfrac{h^{\xi-1}}{\eta} \right)}}$$

$$l_i^y = \frac{1}{1 + \dfrac{w}{b_i \left(\dfrac{h^{\xi-1}}{\eta} \right)}} \left(\frac{a_i w}{\chi_i^y} \right)^{\frac{1}{\phi-1}}$$

and

$$(96) \qquad l_i^m = \frac{1}{1 + \dfrac{w}{b_i \left(\dfrac{h^{\xi-1}}{\eta} \right)}} \left(\frac{(1+g) a_i w}{\chi_i^m} \right)^{\frac{1}{\phi-1}}$$

Then, if $\phi \neq 1$, we have

$$(97) \qquad l_i^y = l_i^m \left(\frac{1}{1+g} \right)^{\frac{1}{\phi-1}}$$

Therefore, young and middle-aged formal work satisfy the same proportion as young and middle-aged informal work. Then, from (62), (97) and (96),

$$
\begin{aligned}
l_i &= \lambda_i l_i^y + \frac{\lambda_i l_i^m}{1+n} = \lambda_i l_i^m \left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}} + \frac{\lambda_i l_i^m}{1+n} \\[2mm]
&= \lambda_i l_i^m \left(\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}} + \frac{1}{1+n}\right) = \lambda_i \left(1 + \frac{(1+g)^{\frac{1}{\phi-1}}}{1+n}\right) \frac{1}{1+\dfrac{w}{b_i\left(\dfrac{h^{\xi-1}}{\eta}\right)}} \left(\frac{a_i w}{\chi_i^m}\right)^{\frac{1}{\phi-1}}
\end{aligned}
\tag{98}
$$

To normalize l_i to 1 - in this steady state - one needs to set

$$
\left(1 + \frac{(1+g)^{\frac{1}{\phi-1}}}{1+n}\right)^{\phi-1} \left(\frac{\lambda_i}{1+\dfrac{w}{b_i\left(\dfrac{h^{\xi-1}}{\eta}\right)}}\right)^{\phi-1} a_i w = \left(\chi_i^m\right)
$$

$$
\chi_i^m = \left(1 + \frac{(1+g)^{\frac{1}{\phi-1}}}{1+n}\right)^{\phi-1} \left(\frac{\lambda_i}{1+\dfrac{w}{b_i\left(\dfrac{h^{\xi-1}}{\eta}\right)}}\right)^{\phi-1} a_i w
$$

From (67), and since $l_i = 1$,

$$
l = \sum_{i=1}^{n_l} a_i l_i \lambda_i = \sum_{i=1}^{n_l} a_i \lambda_i =
\tag{99}
$$

Then, using (97) in (93), we can write

$$
c_i^y = \frac{(1+g) + r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2 + \beta_i + 1\right) r} 2 a_i w l_i^m
$$

$$
l_i^y = l_i^m \left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}
$$

$$(100) \qquad c_i^y = \frac{(1+g)+r\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2+\beta_i+1\right)r} - 2a_i wl_i^m$$

From (41), we have that

$$s_i^y = w_i^y l_i^y + b_i \left(\frac{h^{\xi-1}}{\eta}\right) a_i h_i^y - c_i^y$$

$$(101) \quad = a_i wl_i^m \left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}} + a_i wl_i^m \left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}} - \frac{(1+g)+r\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2+\beta_i+1\right)r} 2a_i wl_i^m$$

$$= \left[\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}} - \frac{(1+g)+r\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2+\beta_i+1\right)r}\right] 2a_i wl_i^m$$

From (42), we have that

$$s_i^m = w_i^m l_i^m + (1+g)b_i \left(\frac{h^{\xi-1}}{\eta}\right) a_i h_i^m + rs_i^y - c_i^m$$

$$= (1+g)a_i wl_i^m + (1+g)a_i l_i^m w + r\left[\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}} - \frac{(1+g)+r\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2+\beta_i+1\right)r}\right] 2a_i wl_i^m - c_i^m$$

$$= \left[1+g+r\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}} - \frac{(1+g)+r\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{(\beta_i^2+\beta_i+1)}\right] 2a_i wl_i^m - \beta_i \frac{(1+g)+r\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{(\beta_i^2+\beta_i+1)} 2a_i wl_i^m$$

$$(102)$$

$$= 2a_i wl_i^m \left(1+g+r\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}} - (1+\beta_i)\frac{(1+g)+r\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{(\beta_i^2+\beta_i+1)}\right)$$

$$= 2a_i wl_i^m \left(1+g+r\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}\right)\left(1-\frac{1+\beta_i}{\beta_i^2+\beta_i+1}\right)$$

Starting from (83),

$$k = (1-\gamma)s$$

$$\overset{(72)}{\cong} (1-\gamma)s^y + \frac{(1-\gamma)s^m}{(1+g)(1+n)}$$

$$\overset{(73)(74)}{\cong} (1-\gamma)\sum_{i=1}^{n_l}\lambda_i\left(s_i^y + \frac{s_i^m}{(1+g)(1+n)}\right)$$

$$\overset{(101)(102)}{\cong} (1-\gamma)$$

$$\sum_{i=1}^{n_l}\lambda_i\left(\begin{array}{c}\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}} - \dfrac{(1+g)+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2+\beta_i+1\right)r} \\[4mm] +\dfrac{1}{(1+g)(1+n)}\left(1+g+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}\right)\left(1-\dfrac{1+\beta_i}{\beta_i^2+\beta_i+1}\right)\end{array}\right)2a_i w l_i^m$$

$$\overset{\beta_i=\beta_j}{\cong} (1-\gamma)2w$$

$$\left(\begin{array}{c}\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}} - \dfrac{(1+g)+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2+\beta_i+1\right)r} \\[4mm] +\dfrac{1}{(1+g)(1+n)}\left(1+g+re\left(\dfrac{e}{1+g}\right)^{\frac{1}{\phi-1}}\right)\left(1-\dfrac{1+\beta_i}{\beta_i^2+\beta_i+1}\right)\end{array}\right)\sum_{i=1}^{n_l}\lambda_i a_i l_i^m$$

$$\overset{(98)}{\cong} \frac{(1-\gamma)2w}{\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}+\dfrac{1}{1+n}}\left(\begin{array}{c}\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}} - \dfrac{(1+g)+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2+\beta_i+1\right)r} \\[4mm] +\dfrac{1}{(1+g)(1+n)}\left(1+g+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}\right)\left(1-\dfrac{1+\beta_i}{\beta_i^2+\beta_i+1}\right)\end{array}\right)\sum_{i=1}^{n_l}a_i l_i$$

$$\stackrel{(99)}{\equiv}\ \frac{(1-\gamma)2}{\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}+\dfrac{1}{1+n}+\dfrac{1}{(1+g)(1+n)}\left[1+g+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}\right]\left(1-\dfrac{1+\beta_i}{\beta_i^2+\beta_i+1}\right)}\left(\begin{array}{c}\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}-\dfrac{(1+g)+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2+\beta_i+1\right)r}\end{array}\right)wl$$

$$\stackrel{(66)(68)}{\equiv}\ (1-\gamma)2\left(\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}+\frac{1}{1+n}\right)^{-1}$$

$$\left(\begin{array}{c}\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}-\dfrac{(1+g)+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2+\beta_i+1\right)r}\\[2em]+\dfrac{1}{(1+g)(1+n)}\left[1+g+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}\right]\left(1-\dfrac{1+\beta_i}{\beta_i^2+\beta_i+1}\right)\end{array}\right)\frac{(1-\alpha)}{(1+g)^{\alpha}(1+n)^{\alpha}}\left(\frac{k}{l}\right)^{\alpha}l$$

Then, the capital labor ratio is then a non linear function of r, and the model parameters,

$$(120)\qquad \frac{k}{l}=\left\{\frac{(1-\gamma)(1-\alpha)2}{(1+g)^{\alpha}(1+n)^{\alpha}}\left(\left(\frac{1}{1+g}\right)^{\frac{1}{\phi-1}}+\frac{1}{1+n}\right)^{-1}\right.$$

$$\left.\left(\begin{array}{c}\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}-\dfrac{(1+g)+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}}{\left(\beta_i^2+\beta_i+1\right)r}\\[2em]+\dfrac{1}{(1+g)(1+n)}\left[1+g+r\left(\dfrac{1}{1+g}\right)^{\frac{1}{\phi-1}}\right]\left(1-\dfrac{1+\beta_i}{\beta_i^2+\beta_i+1}\right)\end{array}\right)\right\}$$

and thus, using (99), we can obtain the steady state value for k,

$$(104) \qquad k = \frac{k}{l}l$$

From (66) and (68), then,

$$(105) \qquad w = (1-\alpha)\left(\frac{k}{(1+g)(1+n)l}\right)^{\alpha}$$

Next, we derive the b_i that ensures that (91) and (92) are satisfied and show that this b_i is common for all i. For this, we depart from equations (91) and (92) and use (58), (59), (95), (96) as well as (86), (97) and (100),

$$b_i = \frac{l_i^y w}{h_i^y\left(\frac{h^{\xi-1}}{\eta}\right)} = \frac{w \dfrac{1}{1+\dfrac{w}{b_i\left(\dfrac{h^{\xi-1}}{\eta}\right)}}\left(\dfrac{a_i w}{\chi_i^y}\right)^{\frac{1}{\phi-1}}}{\left[a_i\dfrac{\left(b_i\left(\dfrac{h^{\xi-1}}{\eta}\right)-w\right)}{\chi_i^y \kappa}\right]^{\frac{1}{\phi-1}}\left(\dfrac{h^{\xi-1}}{\eta}\right)}$$

$$b_i\left(\frac{h^{\xi-1}}{\eta}\right) = \frac{w^{\frac{\phi}{\phi-1}}\dfrac{b_i\left(\dfrac{h^{\xi-1}}{\eta}\right)}{b_i\left(\dfrac{h^{\xi-1}}{\eta}\right)+w}}{\left[\dfrac{\left(b_i\left(\dfrac{h^{\xi-1}}{\eta}\right)-w\right)}{\kappa}\right]^{\frac{1}{\phi-1}}}$$

$$(106) \qquad \left(b_i\left(\frac{h^{\xi-1}}{\eta}\right)-w\right)\left(b_i\left(\frac{h^{\xi-1}}{\eta}\right)+w\right)^{\phi-1} w^{-\phi} = \kappa$$

for all i. The same result is found if departing from

$$(107) \qquad b_i = \frac{l_i^m w}{h_i^m\left(\dfrac{h^{\xi-1}}{\eta}\right)}$$

for all i. Therefore, there exists a b_i that ensures that (91) and (92) are satisfied. In addition, one can observe, that since $w > 0$ by definition, and $\kappa > 0$, $b_i > w$. The remaining steady state values are then as follows,

from (66),

(108)
$$y = \left(\frac{k}{(1+g)(1+n)}\right)^{\alpha} l^{1-\alpha}$$

From (70),

(109)
$$r^k = \alpha \frac{y}{k}(1+g)(1+n)$$

From (71),

(110)
$$r = 1 + (1-\gamma)(r^k - \delta) + \gamma r^* = 1 + \alpha \frac{y}{k}(1+g)(1+n) - \delta$$

Using (105) and (110) to express w and r in terms of the capital labor ratio, (103) and (106) constitute a non-linear system of two equations and two unknowns, b_i and $\frac{\kappa}{l}$. Once we solve for this unknowns, all other variables can be determined. From (73)

(111)
$$s^y = \sum_{i=1}^{n_l} s_i^y \lambda_i$$

From (74),

(112)
$$s^m = \sum_{i=1}^{n_l} s_i^m \lambda_i$$

From (72),

(113)
$$s = s^y + \frac{s^m}{(1+g)(1+n)}$$

From (46),

(114)
$$c^y = \sum_{i=1}^{n_l} c_i^y \lambda_i$$

From (47),

(115)
$$c^m = \sum_{i=1}^{n_l} c_i^m \lambda_i$$

From (48),

(116)
$$c^o = \sum_{i=1}^{n_l} c_i^o \lambda_i$$